Fundamentals of Physics II

Faculty of Physics-Kharazmi University

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اگر همواره مانند گذشته بیندیشید، همیشه همان
چیزهایی را به‌دست می‌آورید که تاکنون کسب کرده‌اید

If you always think the way you've always thought, you'll always get what you've always got.

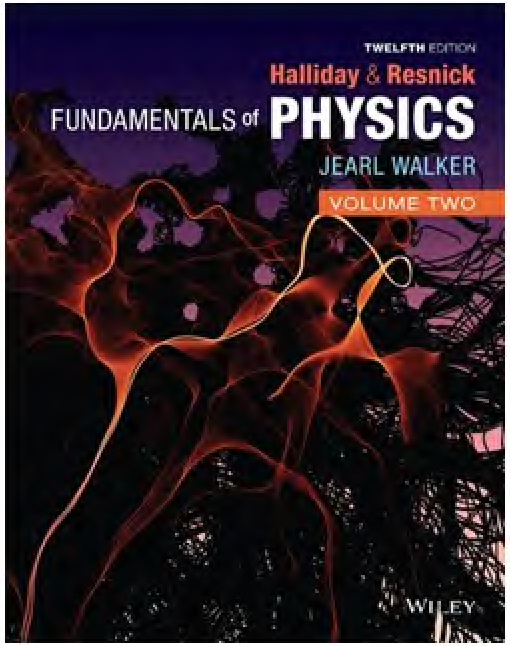
دانشگاه خوارزمی

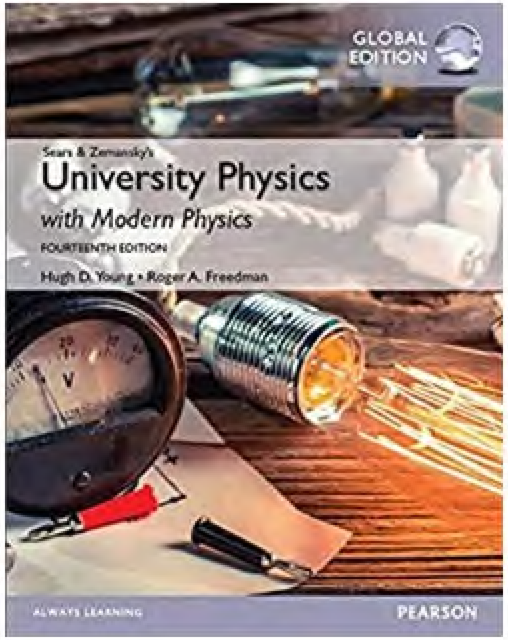
Fundamentals of Physics (12th Ed.)
Halliday, David; Resnick, Robert; Walker, Jearl

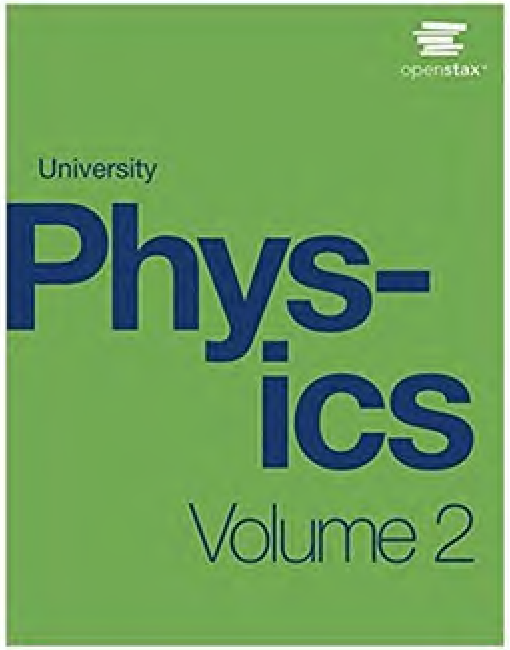
University Physics with Modern Physics (14th Global Ed.)
Hugh D. Young, Roger A. Freedman

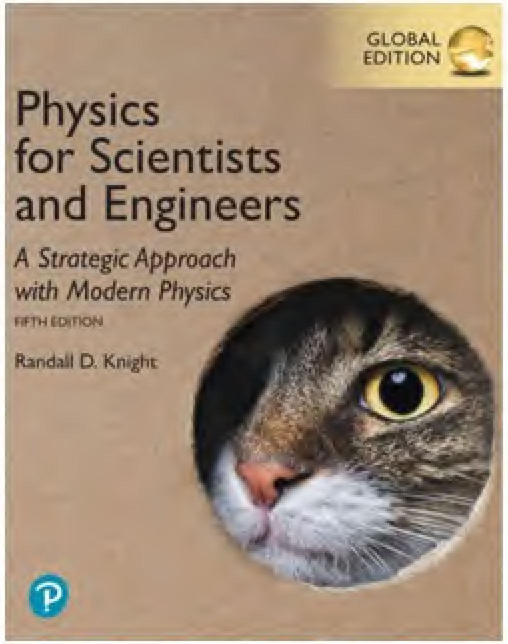
University Physics Volume 2
Samuel J. Ling, Jeff Sanny, William Moebs

P H Y S I C S For Scientists and Engineers, 5e, (2023)
Randall D. Knight









دانشگاه خوارزمی

Lecture 5:

Coordinate Systems Part 2

دانشگاه خوارزمی

- Cartesian coordinate system: Basic Vectors; Differential line, area and volume elements.
- Cylindrical coordinate system: Basic Vectors; Differential line, area and volume elements.
- Spherical coordinate system: Basic Vectors; Differential line, area and volume elements.
- The relationship between the Cartesian coordinate system and the cylindrical coordinate system
- The relationship between the Cartesian coordinate system and the spherical coordinate system

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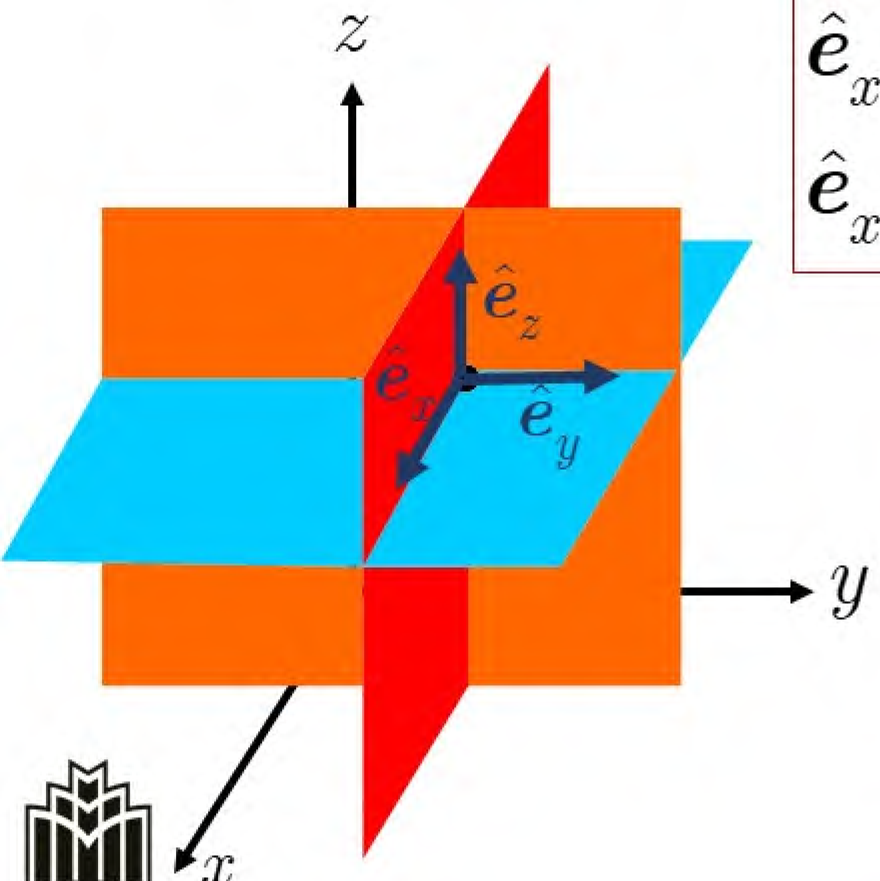
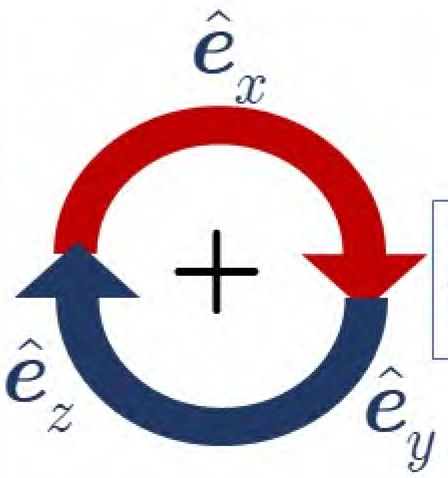
$$\boldsymbol{r} = x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y + z\hat{\boldsymbol{e}}_z$$

$$\boldsymbol{A} = A_x\hat{\boldsymbol{e}}_x + A_y\hat{\boldsymbol{e}}_y + A_z\hat{\boldsymbol{e}}_z$$

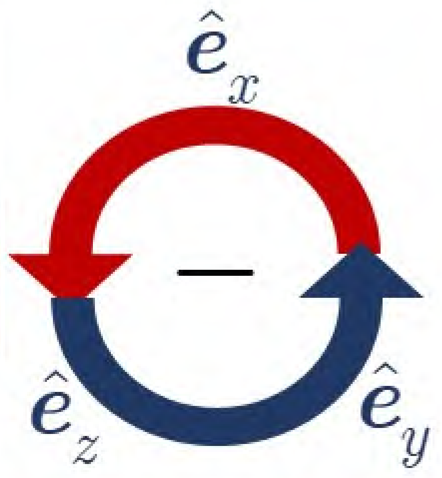
$$A_x = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_x$$
$$A_y = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_y$$
$$A_z = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_z$$

$$\hat{\boldsymbol{e}}_x\cdot\hat{\boldsymbol{e}}_x = \hat{\boldsymbol{e}}_y\cdot\hat{\boldsymbol{e}}_y = \hat{\boldsymbol{e}}_z\cdot\hat{\boldsymbol{e}}_z = 1$$
$$\hat{\boldsymbol{e}}_x\cdot\hat{\boldsymbol{e}}_y = \hat{\boldsymbol{e}}_x\cdot\hat{\boldsymbol{e}}_z = \hat{\boldsymbol{e}}_y\cdot\hat{\boldsymbol{e}}_z = 0$$





$$\hat{\boldsymbol{e}}_x\times\hat{\boldsymbol{e}}_y = \hat{\boldsymbol{e}}_z$$
$$\hat{\boldsymbol{e}}_y\times\hat{\boldsymbol{e}}_x = -\hat{\boldsymbol{e}}_z$$



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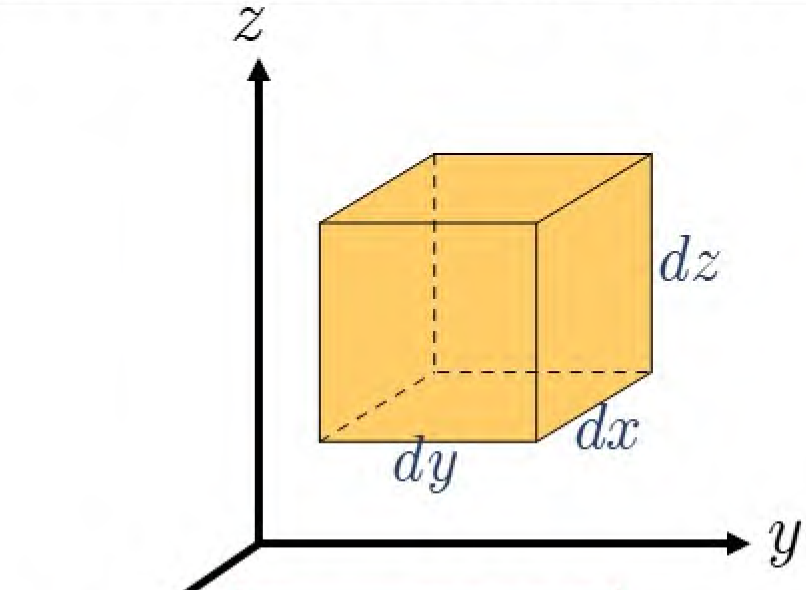
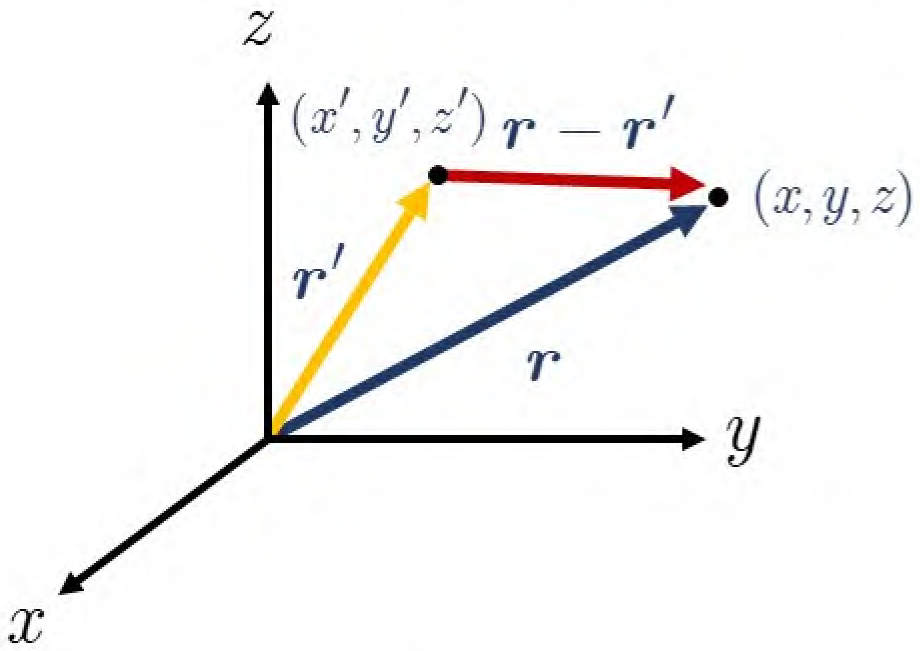


$$dv = dx\,dy\,dz$$

$$d\boldsymbol{l} \equiv d\boldsymbol{r} = dx\,\hat{\boldsymbol{e}}_x + dy\,\hat{\boldsymbol{e}}_y + dz\,\hat{\boldsymbol{e}}_z$$

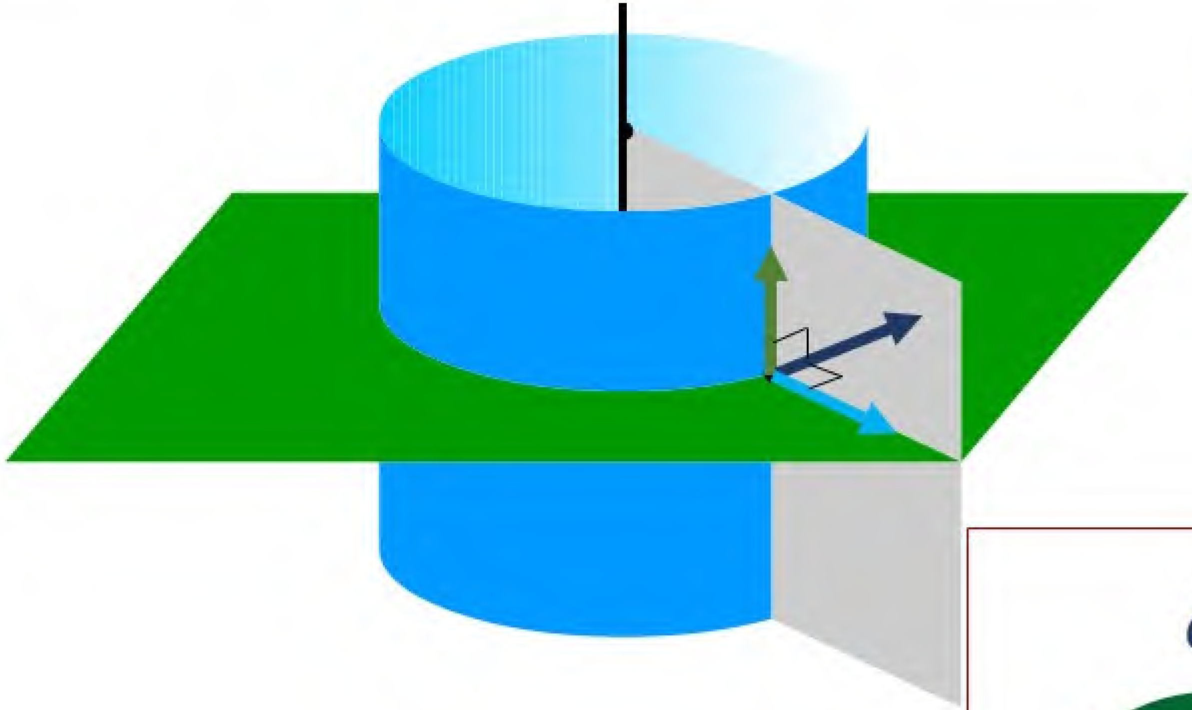
$$da_x = dy\,dz$$
$$da_y = dx\,dz$$
$$da_z = dx\,dy$$



$$\boldsymbol{r} - \boldsymbol{r}' = (x - x')\hat{\boldsymbol{e}}_x + (y - y')\hat{\boldsymbol{e}}_y + (z - z')\hat{\boldsymbol{e}}_z$$

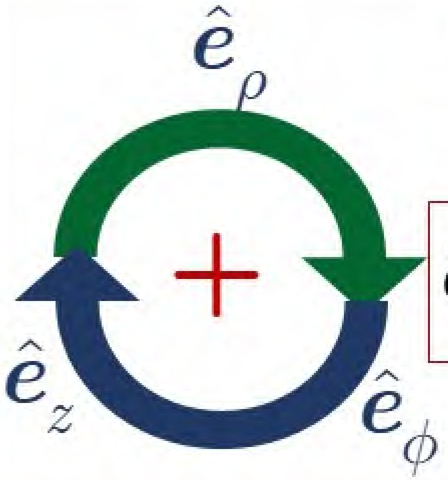
Distance between two points in space

$$d = \left|\boldsymbol{r} - \boldsymbol{r}'\right| = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}$$



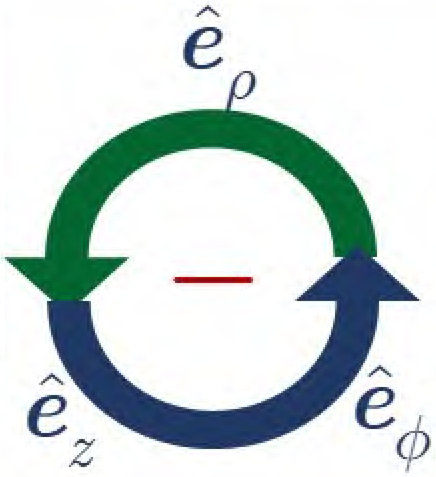
$$\hat{\boldsymbol{e}}_\rho \cdot \hat{\boldsymbol{e}}_\rho = \hat{\boldsymbol{e}}_\phi \cdot \hat{\boldsymbol{e}}_\phi = \hat{\boldsymbol{e}}_z \cdot \hat{\boldsymbol{e}}_z = 1$$

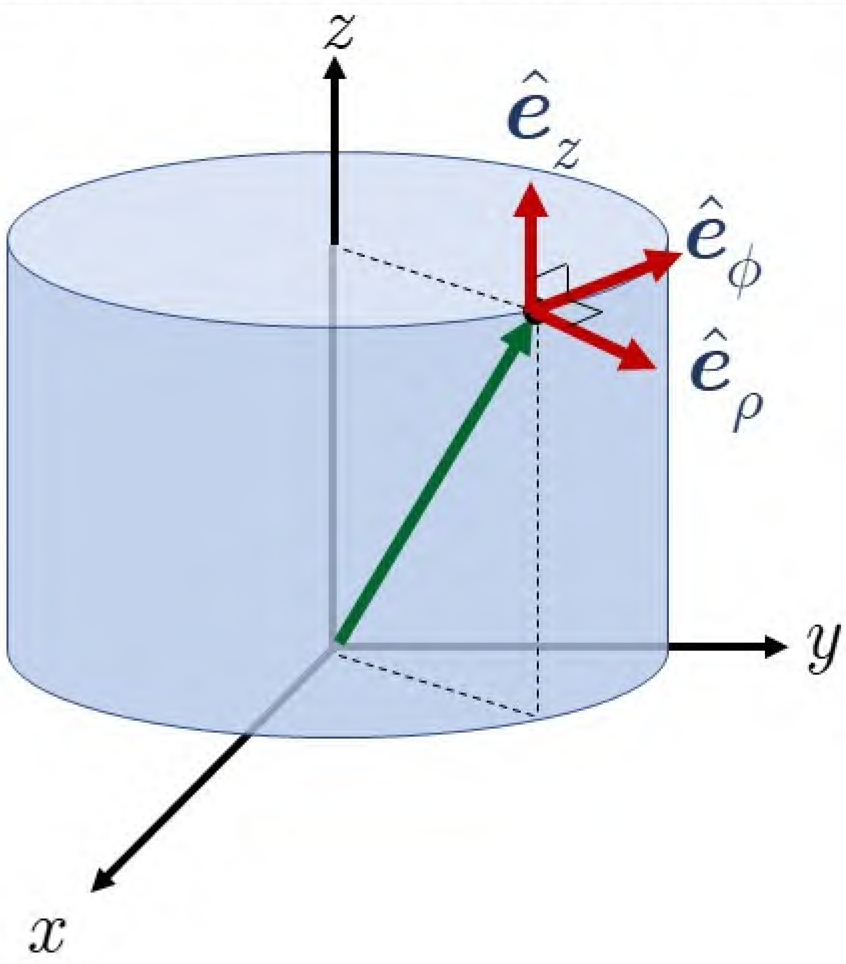
$$\hat{\boldsymbol{e}}_\rho \cdot \hat{\boldsymbol{e}}_z = \hat{\boldsymbol{e}}_\rho \cdot \hat{\boldsymbol{e}}_\phi = \hat{\boldsymbol{e}}_z \cdot \hat{\boldsymbol{e}}_\phi = 0$$



$$\hat{\boldsymbol{e}}_\rho \times \hat{\boldsymbol{e}}_\phi = \hat{\boldsymbol{e}}_z$$

$$\hat{\boldsymbol{e}}_\rho \times \hat{\boldsymbol{e}}_z = -\hat{\boldsymbol{e}}_\phi$$



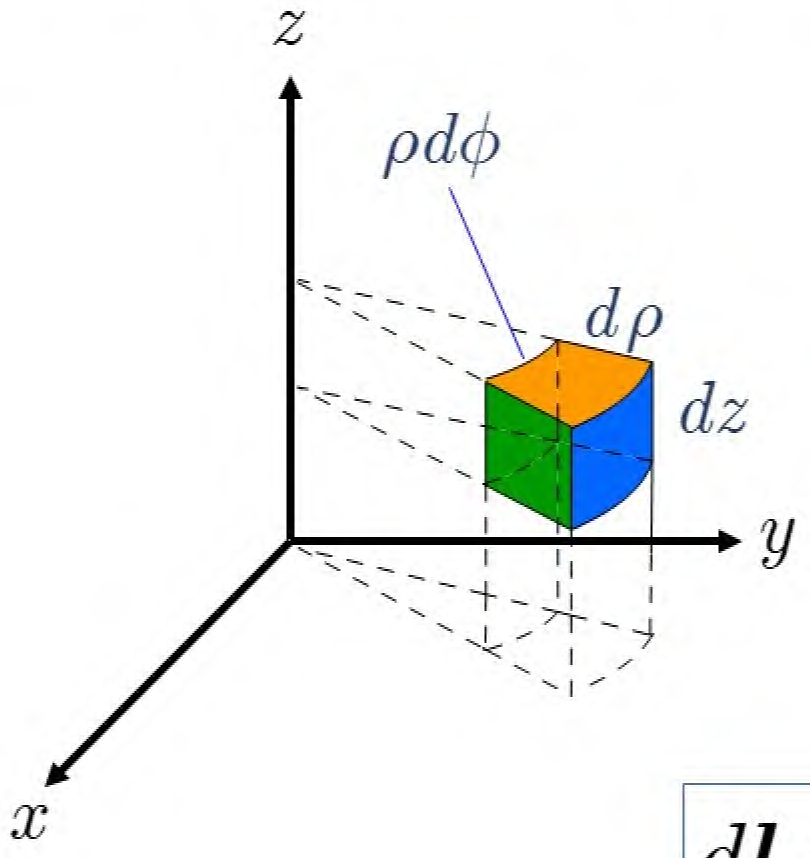


$$\boldsymbol{r} = \rho\hat{\boldsymbol{e}}_\rho + z\hat{\boldsymbol{e}}_z$$

$$|\boldsymbol{r}| = r = \sqrt{\rho^2 + z^2}$$

$$\boldsymbol{A} = A_\rho\hat{\boldsymbol{e}}_\rho + A_\phi\hat{\boldsymbol{e}}_\phi + A_z\hat{\boldsymbol{e}}_z$$

$$|\boldsymbol{A}| = \sqrt{A_\rho^2 + A_\phi^2 + A_z^2}$$



$$da_\rho = \rho d\phi dz$$

$$da_\phi = d\rho dz$$

$$da_z = \rho d\rho d\phi$$

$$dv = \rho d\rho d\phi dz$$

$$d\boldsymbol{l} \equiv d\boldsymbol{r} = d\rho\hat{\boldsymbol{e}}_\rho + \rho d\phi\hat{\boldsymbol{e}}_\phi + dz\hat{\boldsymbol{e}}_z$$

$$d = \left|\boldsymbol{r} - \boldsymbol{r}'\right| = \sqrt{\rho^2 + \rho'^2 - 2\rho\rho'\cos(\phi - \phi') + (z - z')^2}$$

$$x = \rho\cos\phi$$
$$y = \rho\sin\phi$$
$$z = z$$

$$\rho = \sqrt{x^2 + y^2}$$
$$\phi = \tan^{-1}\frac{y}{x}$$
$$z = z$$

$$\hat{e}_x = \cos\phi\,\hat{e}_\rho - \sin\phi\,\hat{e}_\phi$$
$$\hat{e}_y = \sin\phi\,\hat{e}_\rho + \cos\phi\,\hat{e}_\phi$$
$$\hat{e}_z = \hat{e}_z$$

$$\hat{e}_\rho = \cos\phi\,\hat{e}_x + \sin\phi\,\hat{e}_y$$
$$\hat{e}_\phi = -\sin\phi\,\hat{e}_x + \cos\phi\,\hat{e}_y$$
$$\hat{e}_z = \hat{e}_z$$

دانشگاه خوارزمی

Conversion vectors in cylindrical to cartesian coordinates

$$\boldsymbol{A} = A_\rho \hat{\boldsymbol{e}}_\rho + A_\phi \hat{\boldsymbol{e}}_\phi + A_z \hat{\boldsymbol{e}}_z$$

$$A_x = \boldsymbol{A} \cdot \hat{\boldsymbol{e}}_x = A_\rho \hat{\boldsymbol{e}}_\rho \cdot \hat{\boldsymbol{e}}_x + A_\phi \hat{\boldsymbol{e}}_\phi \cdot \hat{\boldsymbol{e}}_x + A_z \hat{\boldsymbol{e}}_z \cdot \hat{\boldsymbol{e}}_x$$
$$A_y = \boldsymbol{A} \cdot \hat{\boldsymbol{e}}_y = A_\rho \hat{\boldsymbol{e}}_\rho \cdot \hat{\boldsymbol{e}}_y + A_\phi \hat{\boldsymbol{e}}_\phi \cdot \hat{\boldsymbol{e}}_y + A_z \hat{\boldsymbol{e}}_z \cdot \hat{\boldsymbol{e}}_y$$
$$A_z = \boldsymbol{A} \cdot \hat{\boldsymbol{e}}_z = A_\rho \hat{\boldsymbol{e}}_\rho \cdot \hat{\boldsymbol{e}}_z + A_\phi \hat{\boldsymbol{e}}_\phi \cdot \hat{\boldsymbol{e}}_z + A_z \hat{\boldsymbol{e}}_z \cdot \hat{\boldsymbol{e}}_z$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_\rho \\ A_\phi \\ A_z \end{pmatrix}$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \frac{x}{\sqrt{x^2+y^2}} & \frac{-y}{\sqrt{x^2+y^2}} & 0 \\ \frac{y}{\sqrt{x^2+y^2}} & \frac{x}{\sqrt{x^2+y^2}} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_\rho \\ A_\phi \\ A_z \end{pmatrix}$$

دانشگاه خوارزمی

Conversion vectors in cartesian to cylindrical coordinates

$$\begin{pmatrix} A_\rho \\ A_\phi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

دانشگاه خوارزمی

The vector $\boldsymbol{A} = \hat{\boldsymbol{e}}_x + x\hat{\boldsymbol{e}}_y + (x+y)\hat{\boldsymbol{e}}_z$ is given in the Cartesian coordinate system. Find the components of this vector in the cylindrical coordinate system at the point (-1, 2, 3)

$$\rho = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + 2^2} = 2.24$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{2}{-1} = 116.57^\circ$$

$$z = 3$$

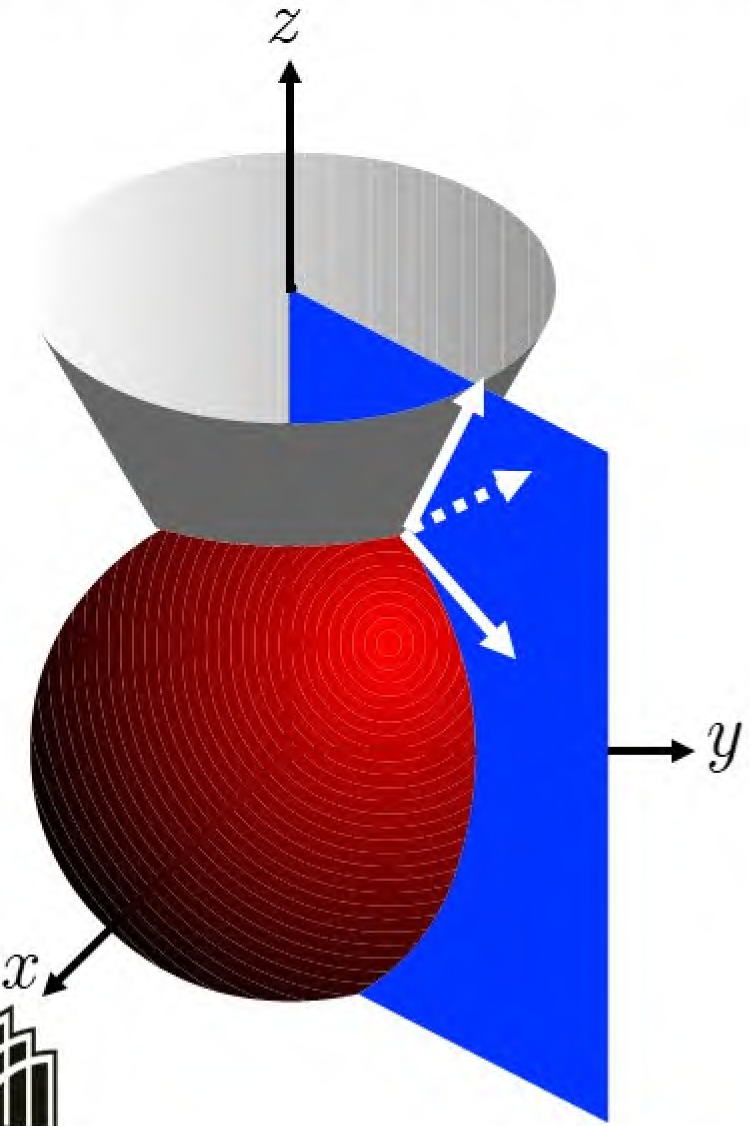
$$\begin{pmatrix} A_\rho \\ A_\phi \\ A_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ x \\ x+y \end{pmatrix}$$

$$A_\rho = \cos\phi + x\sin\phi$$
$$A_\phi = -\sin\phi + x\cos\phi$$
$$A_z = x + y$$

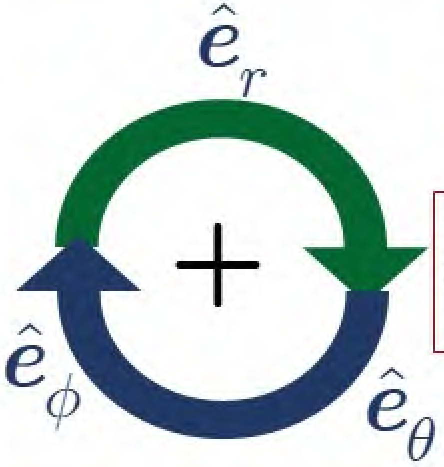
$$A_\rho = \cos\phi + \rho\cos\phi\sin\phi = -1.34$$
$$A_\phi = -\sin\phi + \rho\cos\phi\cos\phi = -0.45$$
$$A_z = \rho\cos\phi + \rho\sin\phi = 1$$

Therefore, at the point (-1, 2, 3), the above vector is equal to

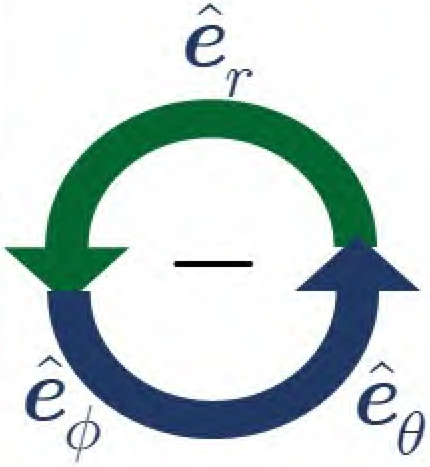
$$\left(A_\rho, A_\phi, A_z\right) = \left(-1.34, -0.45, 1\right)$$

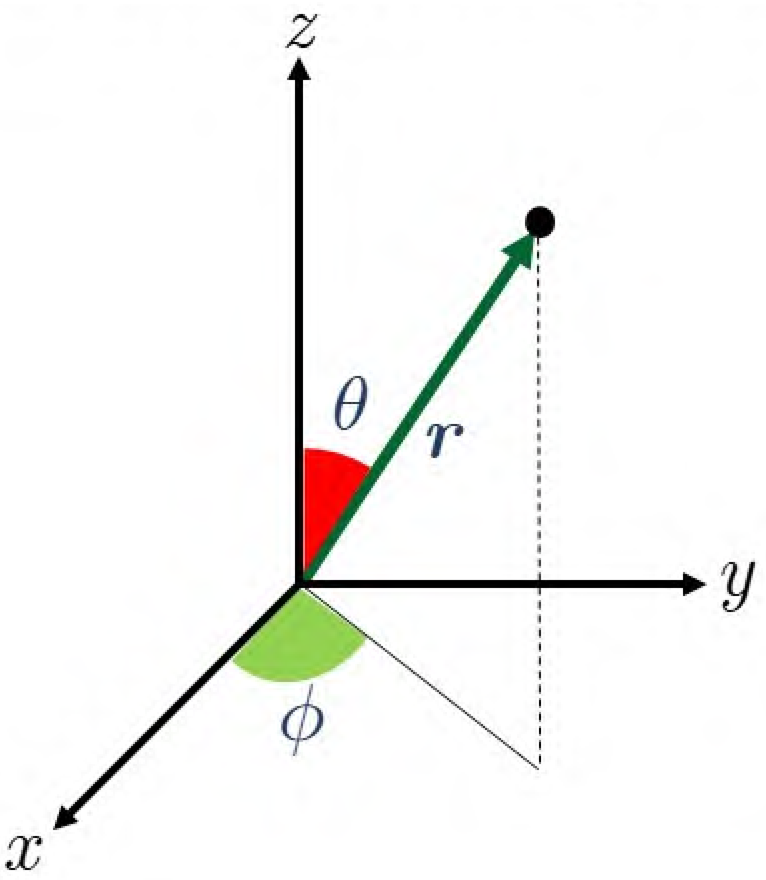


$$\hat{\boldsymbol{e}}_r \cdot \hat{\boldsymbol{e}}_r = \hat{\boldsymbol{e}}_\theta \cdot \hat{\boldsymbol{e}}_\theta = \hat{\boldsymbol{e}}_\phi \cdot \hat{\boldsymbol{e}}_\phi = 1$$
$$\hat{\boldsymbol{e}}_r \cdot \hat{\boldsymbol{e}}_\theta = \hat{\boldsymbol{e}}_r \cdot \hat{\boldsymbol{e}}_\phi = \hat{\boldsymbol{e}}_\theta \cdot \hat{\boldsymbol{e}}_\phi = 0$$



$$\hat{\boldsymbol{e}}_r \times \hat{\boldsymbol{e}}_\theta = \hat{\boldsymbol{e}}_\phi$$
$$\hat{\boldsymbol{e}}_r \times \hat{\boldsymbol{e}}_\phi = -\hat{\boldsymbol{e}}_\theta$$





$$\boldsymbol{r} = r\hat{\boldsymbol{e}}_r$$

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

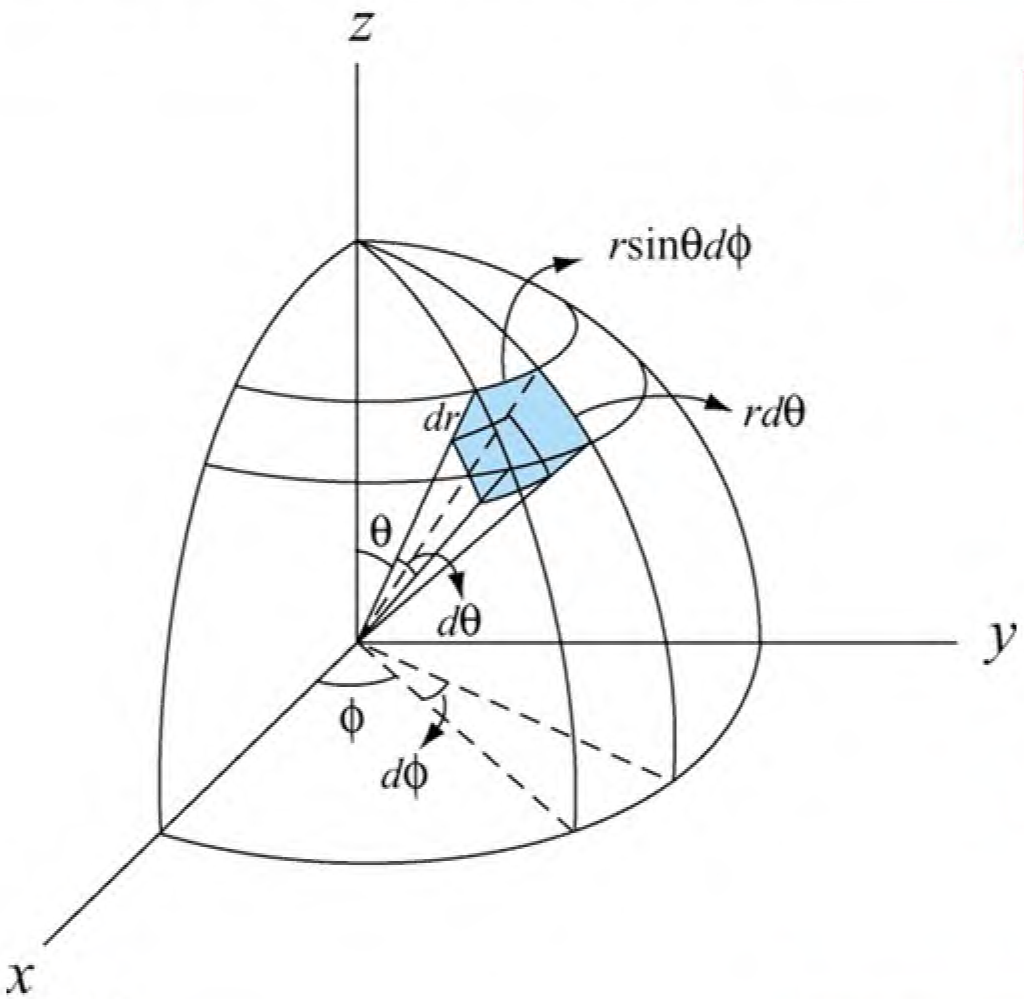
$$r = \sqrt{x^2 + y^2 + z^2}$$

$$\theta = \tan^{-1}\frac{\sqrt{x^2+y^2}}{z} = \cos^{-1}\frac{z}{\sqrt{x^2+y^2+z^2}}$$

$$\phi = \tan^{-1}\frac{y}{x}$$

$$d = \left|\boldsymbol{r} - \boldsymbol{r}'\right| = \sqrt{r^2 + r'^2 - 2rr'\cos\theta\cos\theta' - 2rr'\sin\theta\sin\theta'\cos(\phi - \phi')}$$

دانشگاه خوارزمی



$$dv = r^2 \sin\theta dr d\theta d\phi$$

$$da_r = r^2 \sin\theta d\theta d\phi$$

$$da_\theta = r \sin\theta dr d\phi$$

$$da_\phi = r d\theta dr$$

$$d\boldsymbol{l} \equiv d\boldsymbol{r} = dr\hat{\boldsymbol{e}}_r + r d\theta \hat{\boldsymbol{e}}_\theta + r\sin\theta d\phi \hat{\boldsymbol{e}}_\phi$$

Conversion vectors in spherical to cartesian coordinates

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ \cos\theta & -\sin\theta & \circ \end{pmatrix} \begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix}$$

دانشگاه خوارزمی

Conversion vectors in cartesian to spherical coordinates

$$\begin{pmatrix} A_r \\ A_\theta \\ A_\phi \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & \sin\theta\sin\phi & \cos\theta \\ \cos\theta\cos\phi & \cos\theta\sin\phi & -\sin\theta \\ -\sin\phi & \cos\phi & \circ \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

دانشگاه خوارزمی

شاد و مهربان باشید

Be happy and kind