Fundamentals of Physics II

Faculty of Physics-Kharazmi University

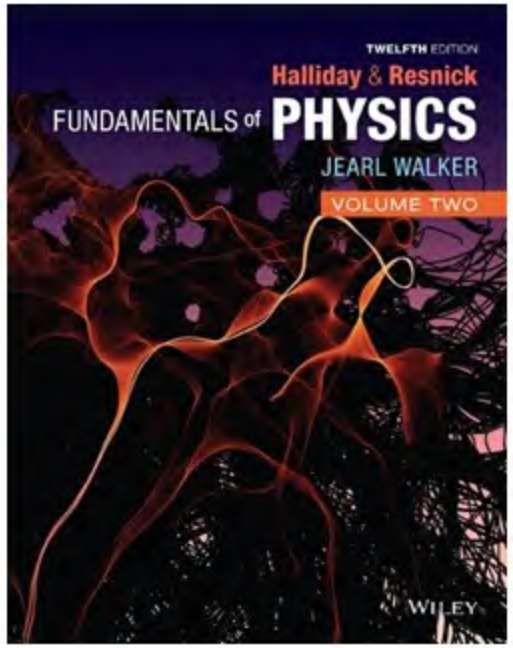
Dr. Faramarz Kanjouri

Spring 2025

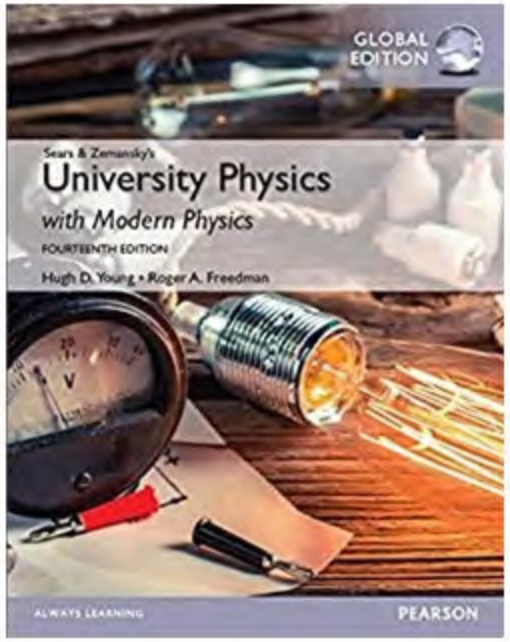
اگر همواره مانند گذشته بیندیشید، همیشه همان
چیزهایی را به‌دست می‌آورید که تاکنون کسب کرده‌اید

If you always think the way you've always thought, you'll always get what you've always got.

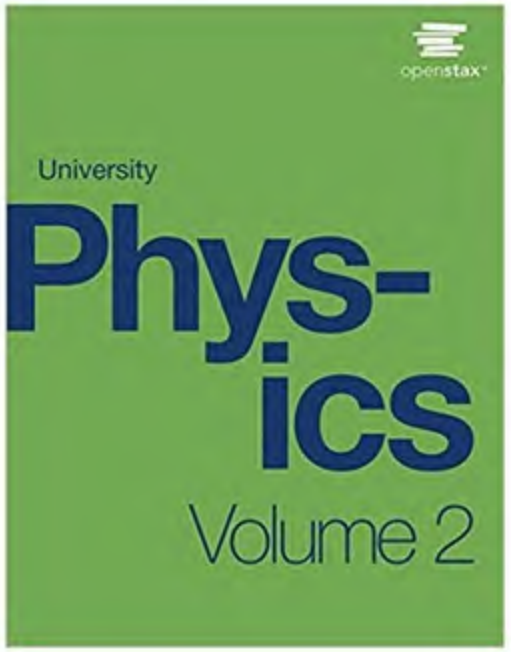
Fundamentals of Physics (12th Ed.)
Halliday, David; Resnick, Robert; Walker, Jearl



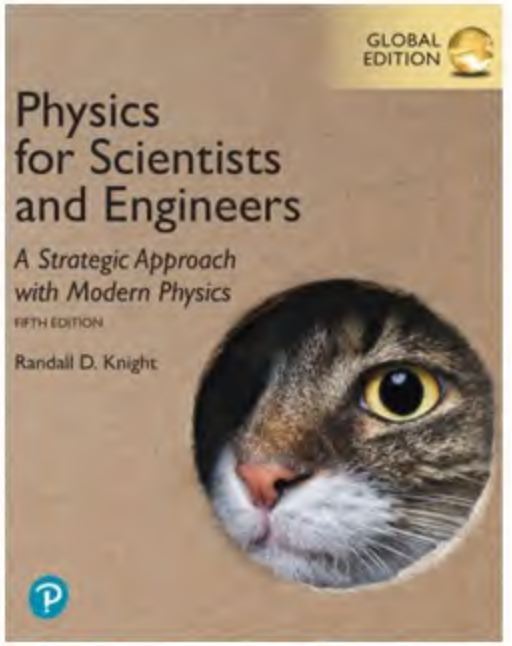
University Physics with Modern Physics (14th Global Ed.)
Hugh D. Young, Roger A. Freedman



University Physics Volume 2
Samuel J. Ling, Jeff Sanny, William Moebs



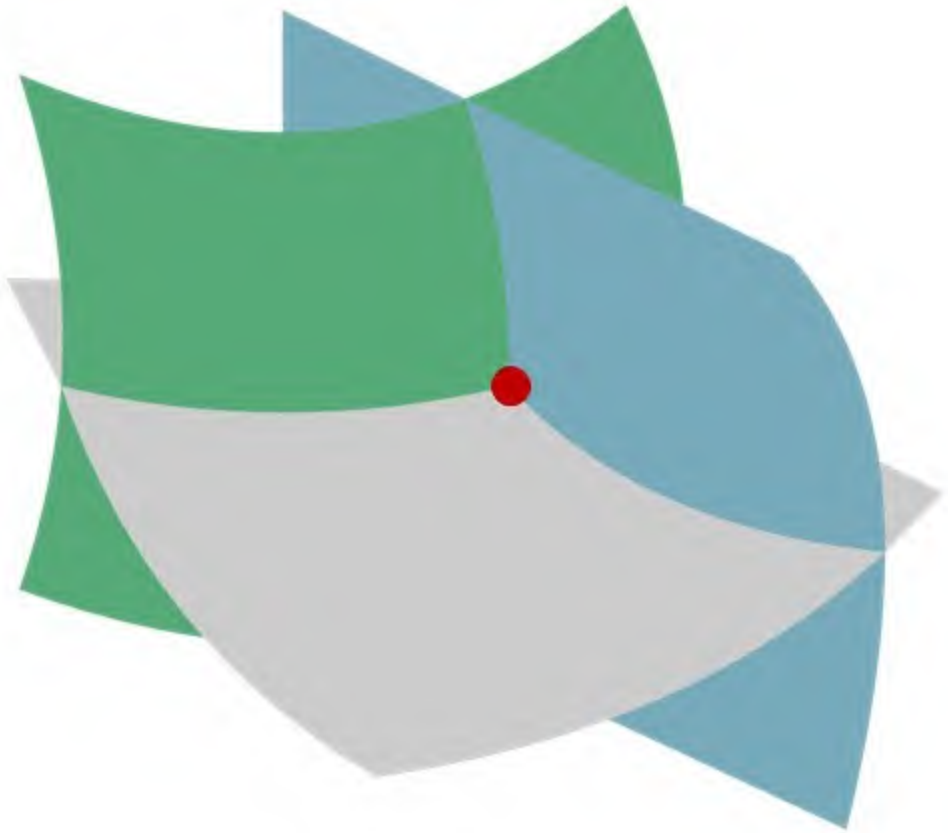
P H Y S I C S For Scientists and Engineers, 5e, (2023)
Randall D. Knight



Lecture 4:

Coordinate Systems Part 1

- What is a coordinate system?
- A general introduction to Cartesian, spherical, and cylindrical coordinate systems
- Line and Surface elements in two dimensional cartesian system
- Line and Surface elements in polar coordinate system.
- The relationship between polar and Cartesian coordinate systems

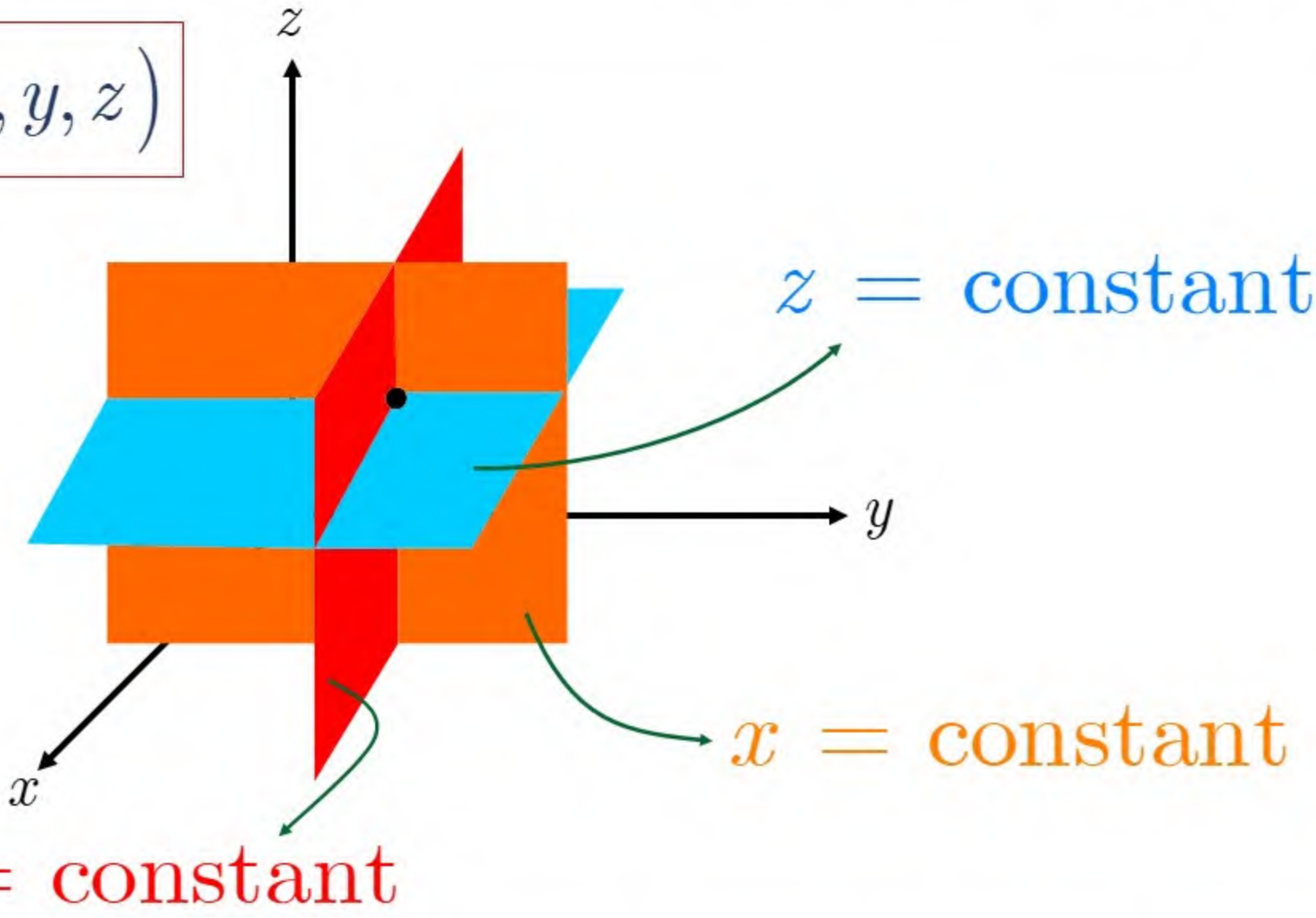


$$u_1 = \text{constant}$$

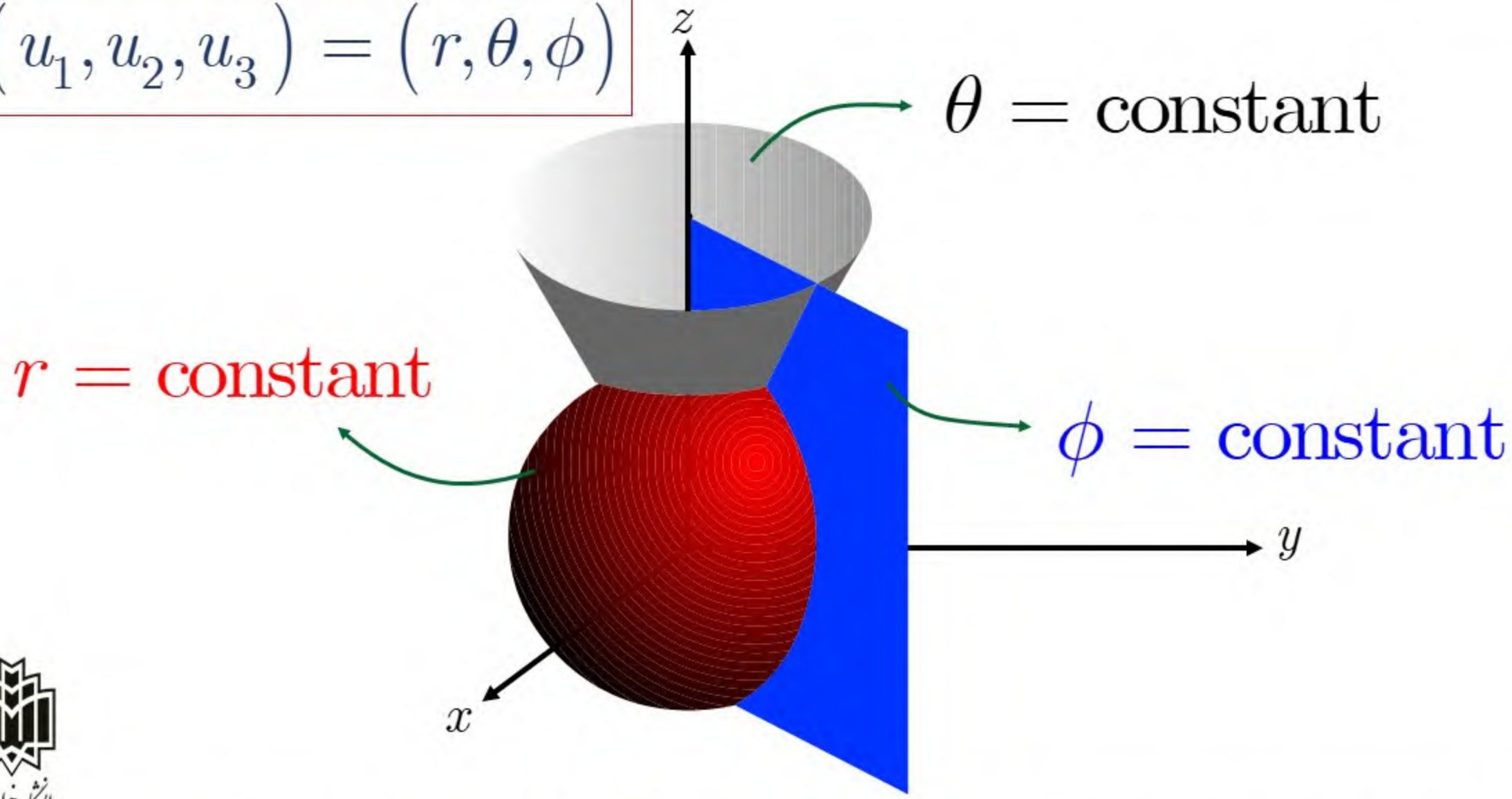
$$u_2 = \text{constant}$$

$$u_3 = \text{constant}$$

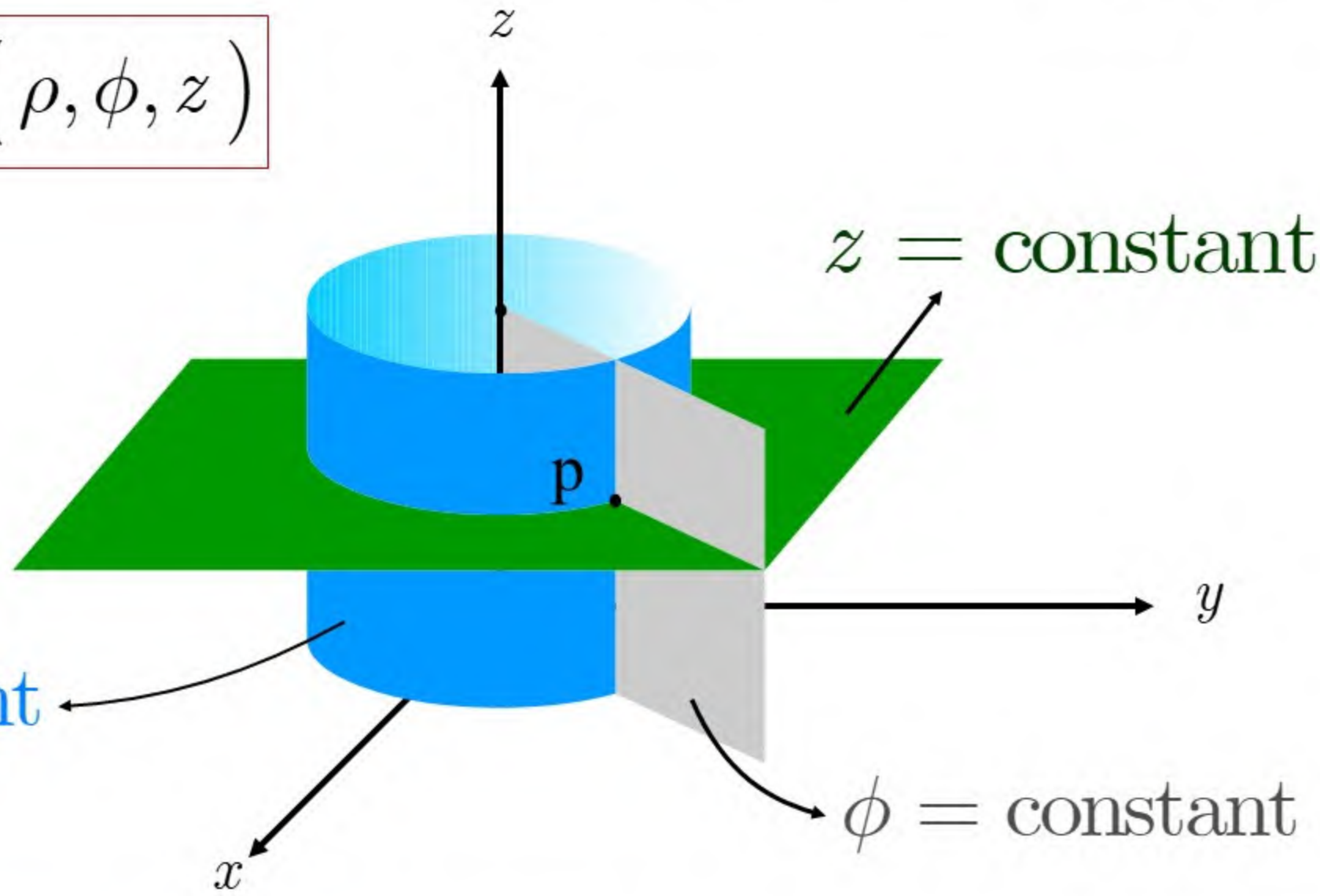
$$\left(u_1, u_2, u_3\right) = \left(x, y, z\right)$$

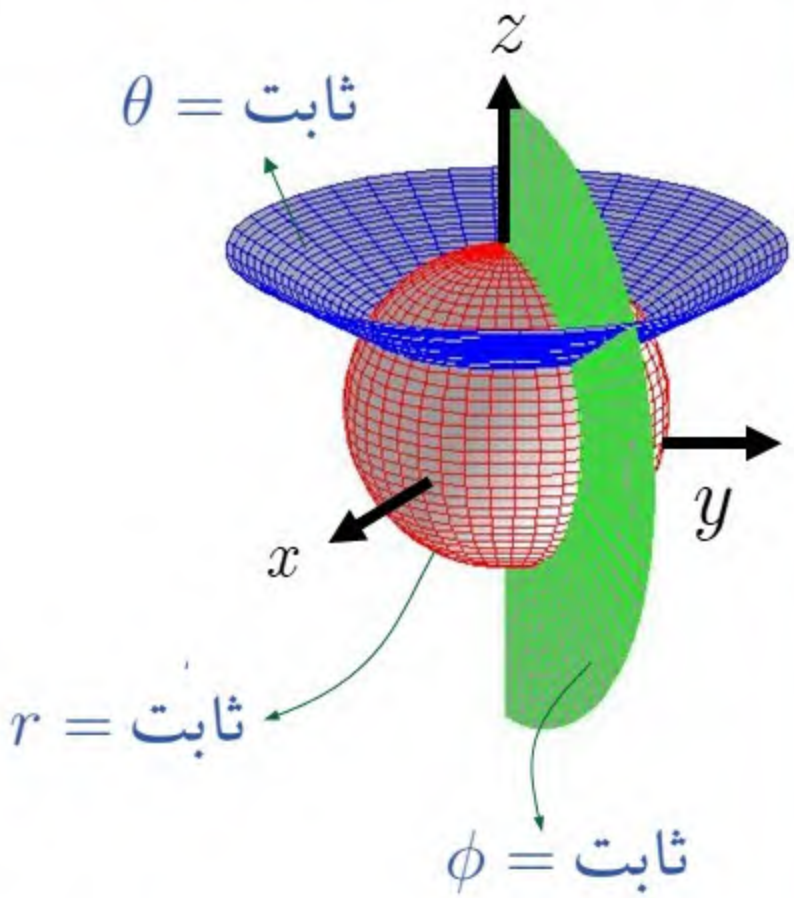


$$\left(u_1, u_2, u_3\right) = \left(r, \theta, \phi\right)$$

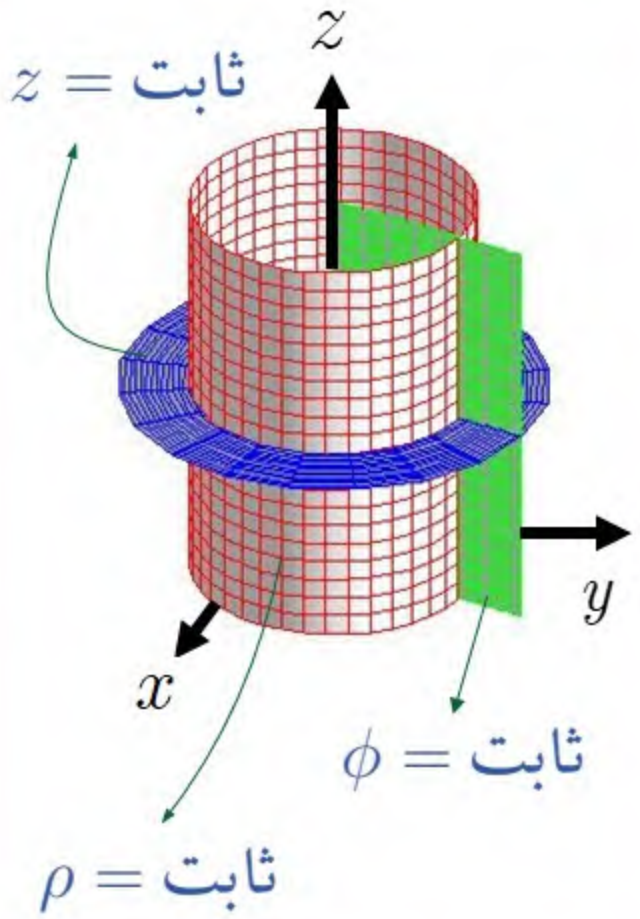


$$\left(u_1, u_2, u_3\right) = \left(\rho, \phi, z\right)$$

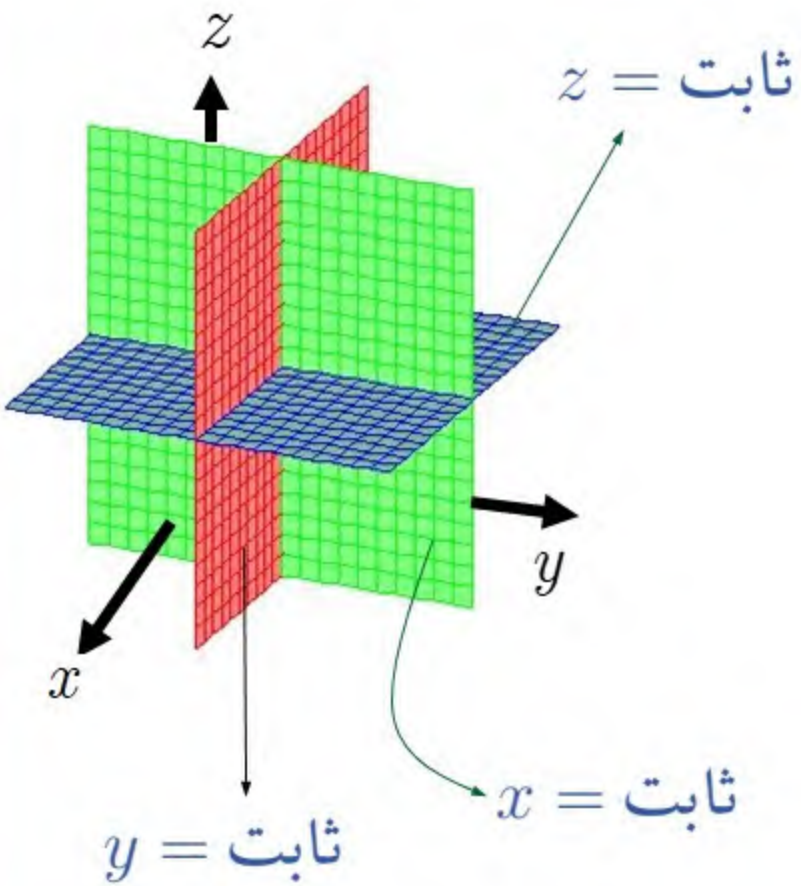




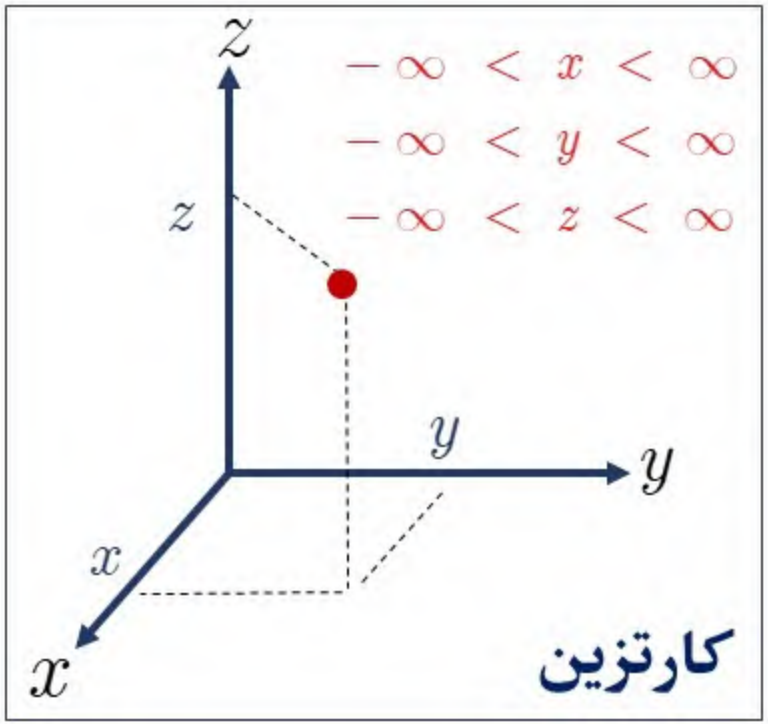
$$\left(u_1, u_2, u_3\right) = \left(r, \theta, \phi\right)$$

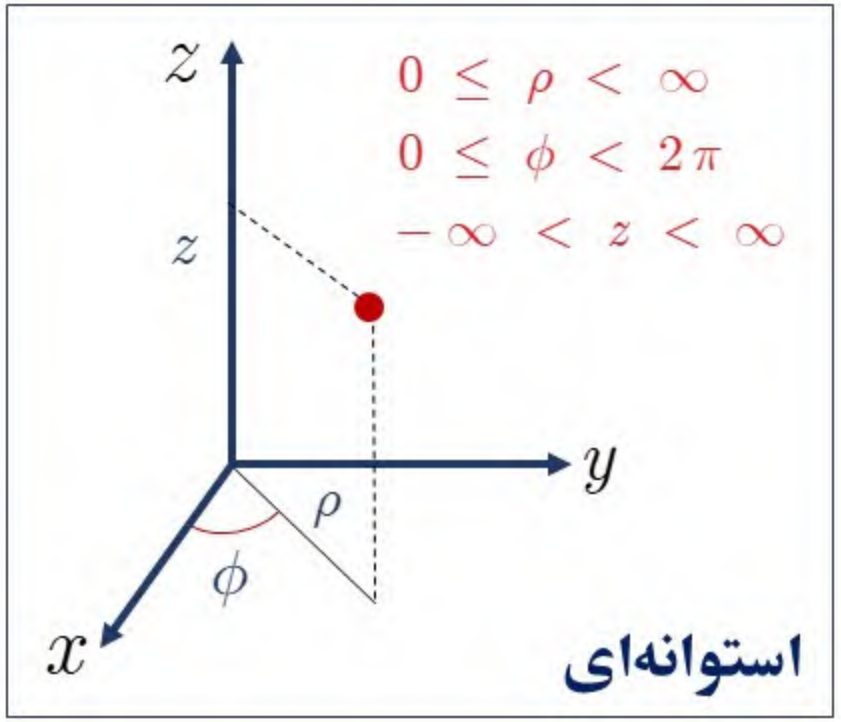


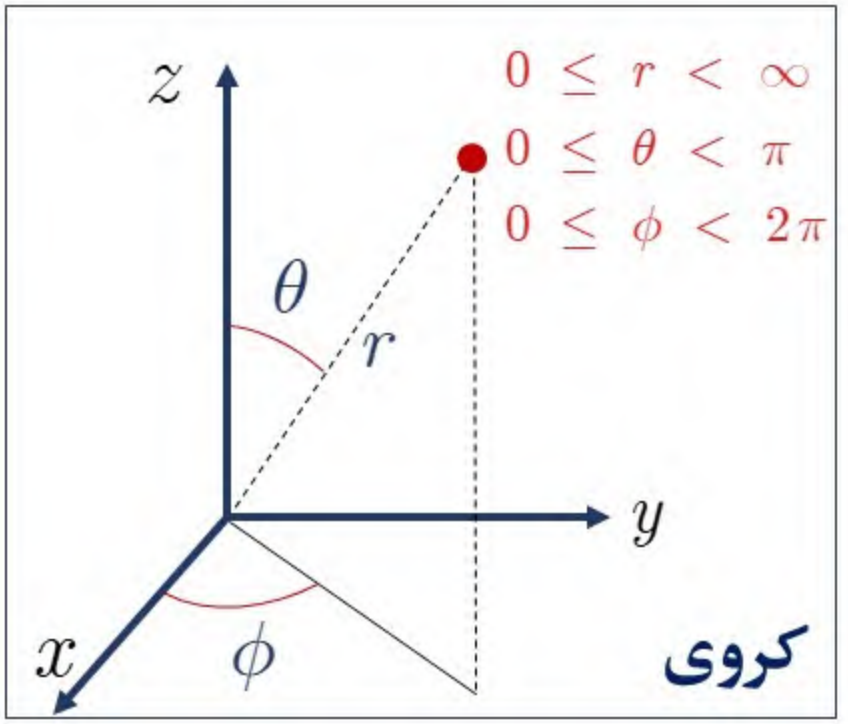
$$\left(u_1, u_2, u_3\right) = \left(\rho, \phi, z\right)$$

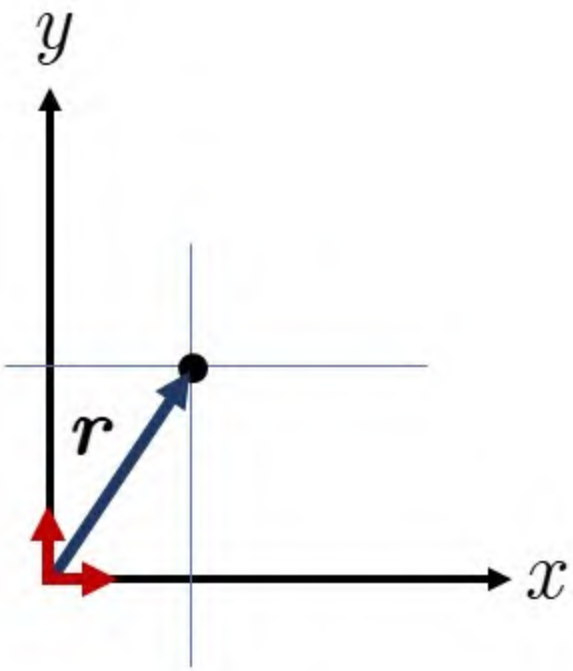


$$\left(u_1, u_2, u_3\right) = \left(x, y, z\right)$$

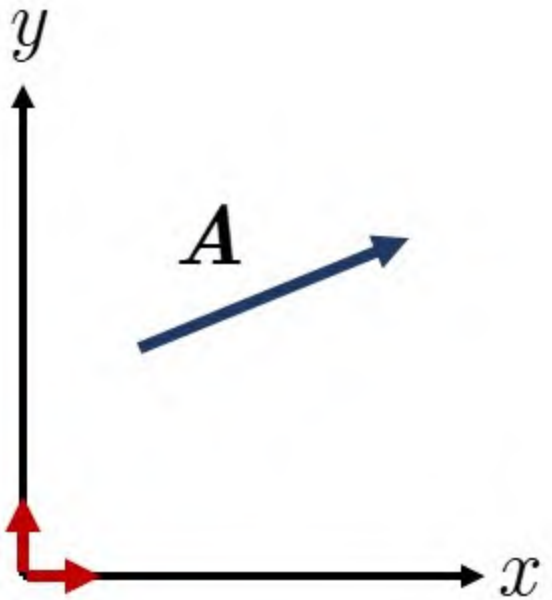

$-\infty < x < \infty$
$-\infty < y < \infty$
$-\infty < z < \infty$
z
y
x
کارتزین


$0 \le \rho < \infty$
$0 \le \phi < 2\pi$
$-\infty < z < \infty$
z
ρ
ϕ
y
x
استوانه‌ای


$0 \le r < \infty$
$0 \le \theta < \pi$
$0 \le \phi < 2\pi$
z
θ
r
y
x
ϕ
کروی



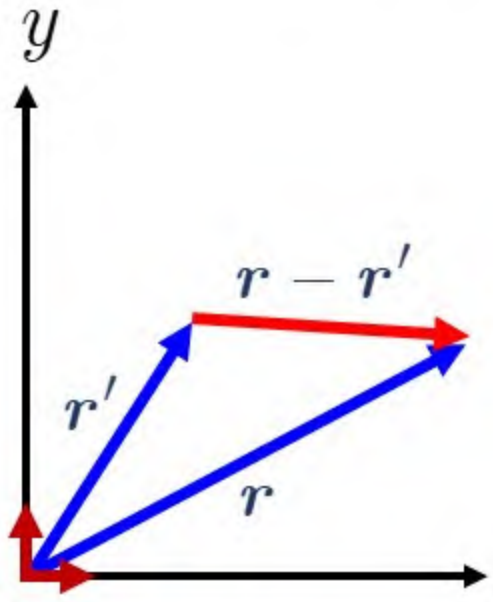
$$\boldsymbol{r} = x\hat{\boldsymbol{e}}_x + y\hat{\boldsymbol{e}}_y$$



$$\boldsymbol{A} = A_x\hat{\boldsymbol{e}}_x + A_y\hat{\boldsymbol{e}}_y$$

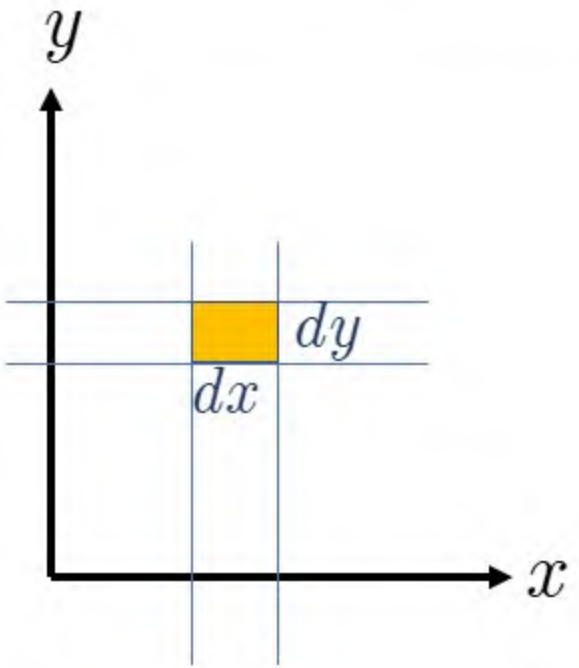
$$A_x = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_x$$

$$A_y = \boldsymbol{A}\cdot\hat{\boldsymbol{e}}_y$$



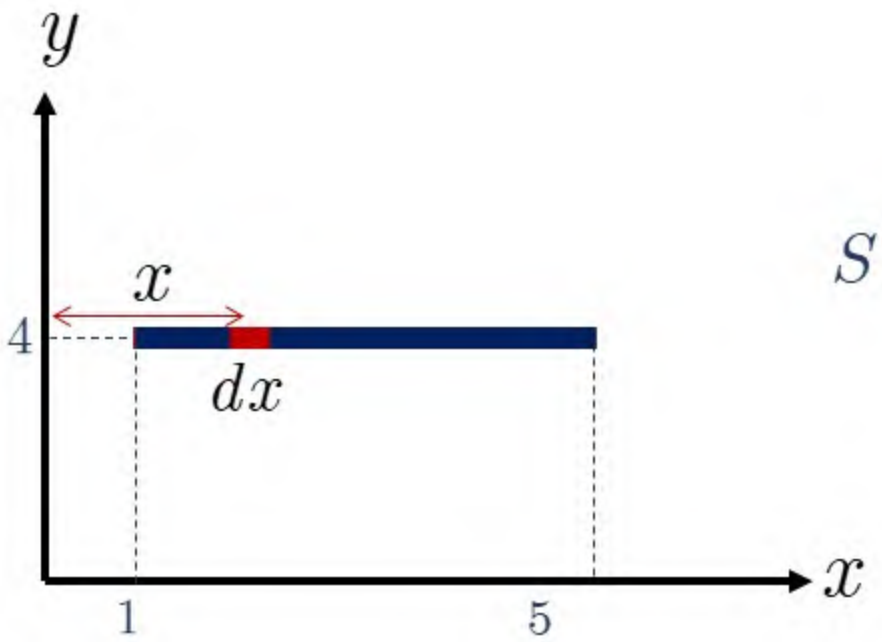
$$\boldsymbol{r} - \boldsymbol{r}' = (x - x')\hat{\boldsymbol{e}}_x + (y - y')\hat{\boldsymbol{e}}_y$$

$$d = \left|\boldsymbol{r} - \boldsymbol{r}'\right| = \sqrt{(x - x')^2 + (y - y')^2}$$



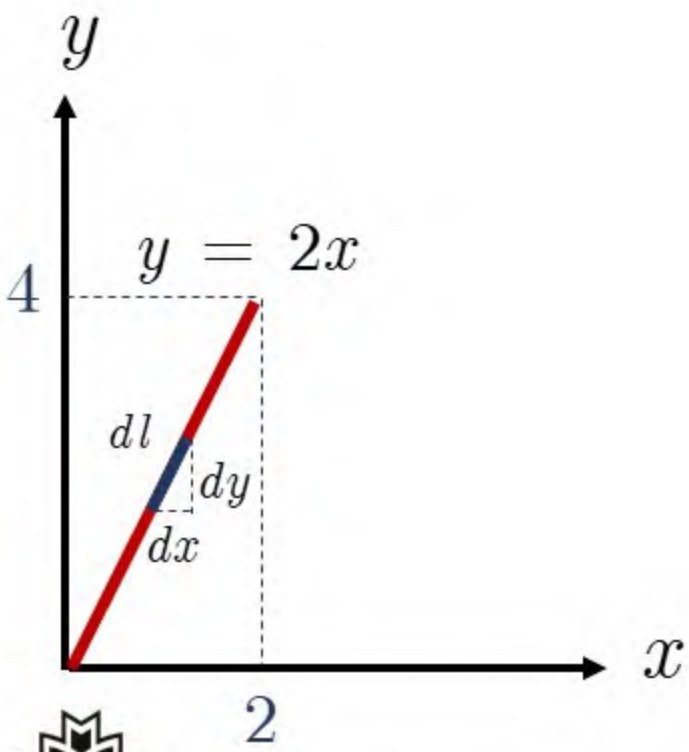
$$d\boldsymbol{l} \equiv d\boldsymbol{r} = dx\hat{\boldsymbol{e}}_x + dy\hat{\boldsymbol{e}}_y$$

$$da = dxdy$$



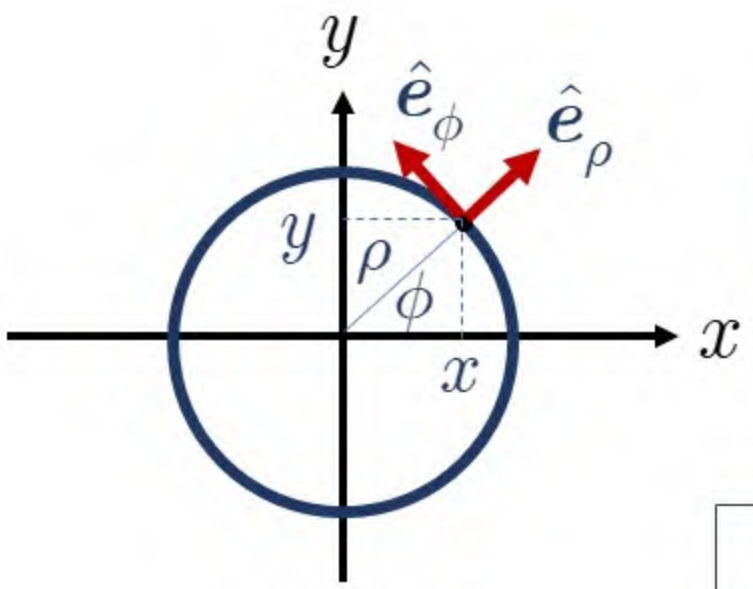
$$S = \int_1^5 dx = x\Big|_1^5 = 5 - 1 = 4$$

$$L = \int_{x=0,y=0}^{x=2,y=4} dl = \int_{x=0,y=0}^{x=2,y=4} \sqrt{dx^2 + dy^2} = \int_{x=0,y=0}^{x=2,y=4} dx \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$$



$$L = \int_{x=0}^{x=2} dx \sqrt{1 + \left(2\right)^2}$$

$$L = \sqrt{5} \int_{x=0}^{x=2} dx = 2\sqrt{5}$$

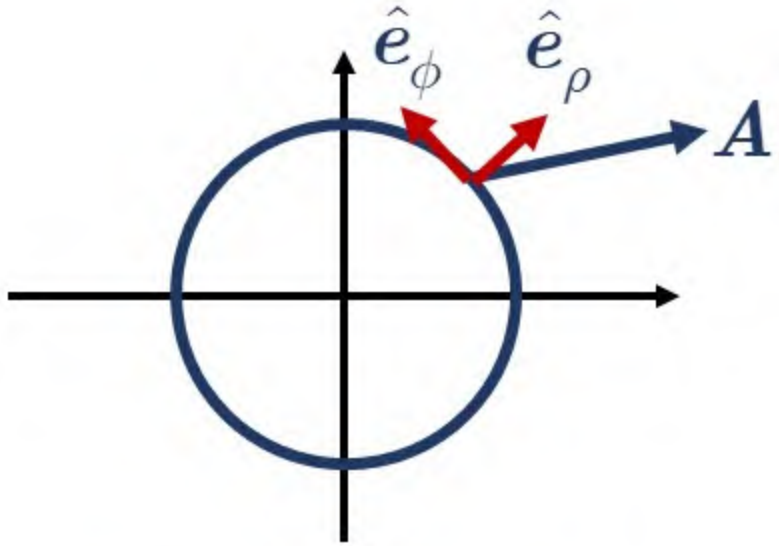


$$\boldsymbol{r} = \rho \hat{\boldsymbol{e}}_\rho$$

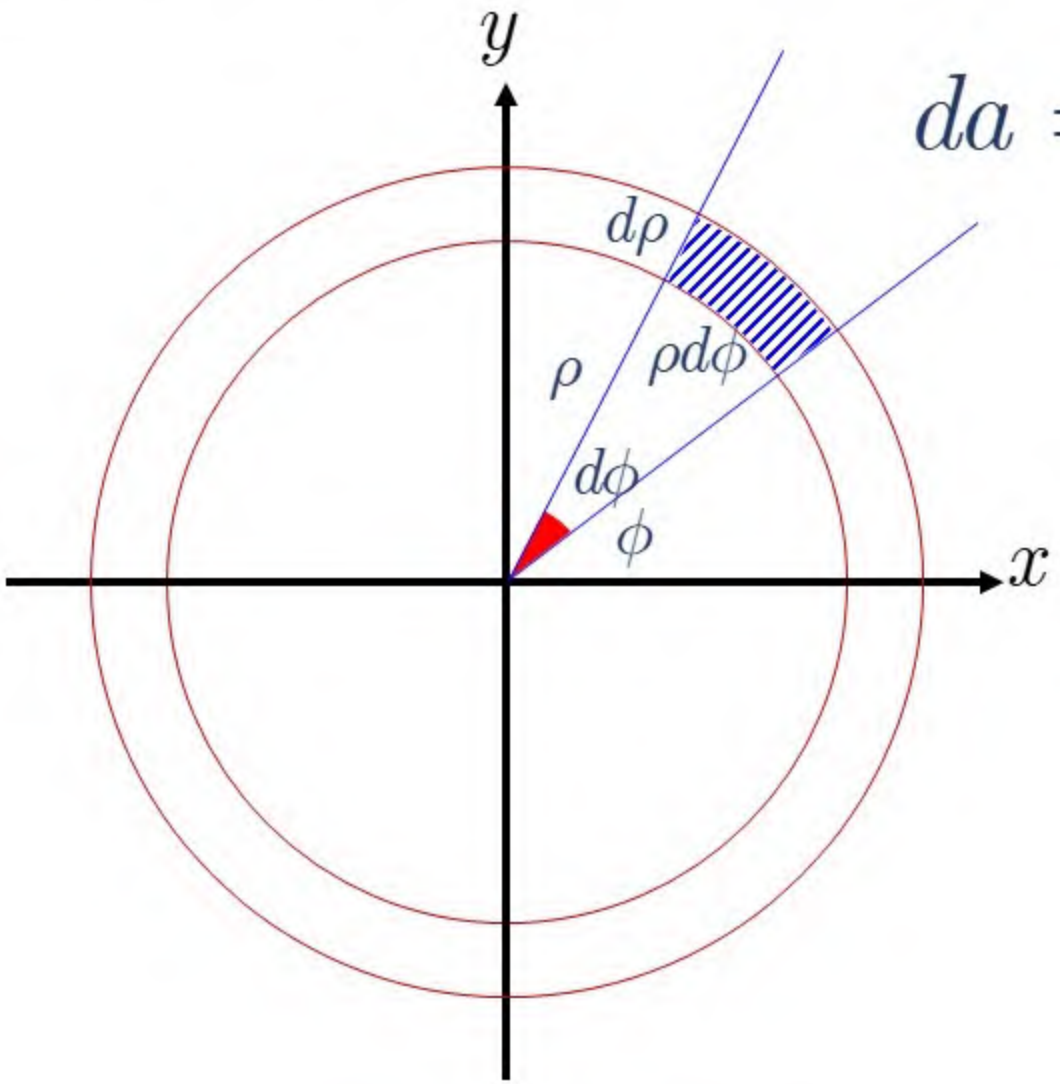
$$\begin{aligned} x &= \rho \cos\phi \\ y &= \rho \sin\phi \end{aligned}$$

$$\begin{aligned} \rho &= \sqrt{x^2 + y^2} \\ \phi &= \tan^{-1} \frac{y}{x} \end{aligned}$$

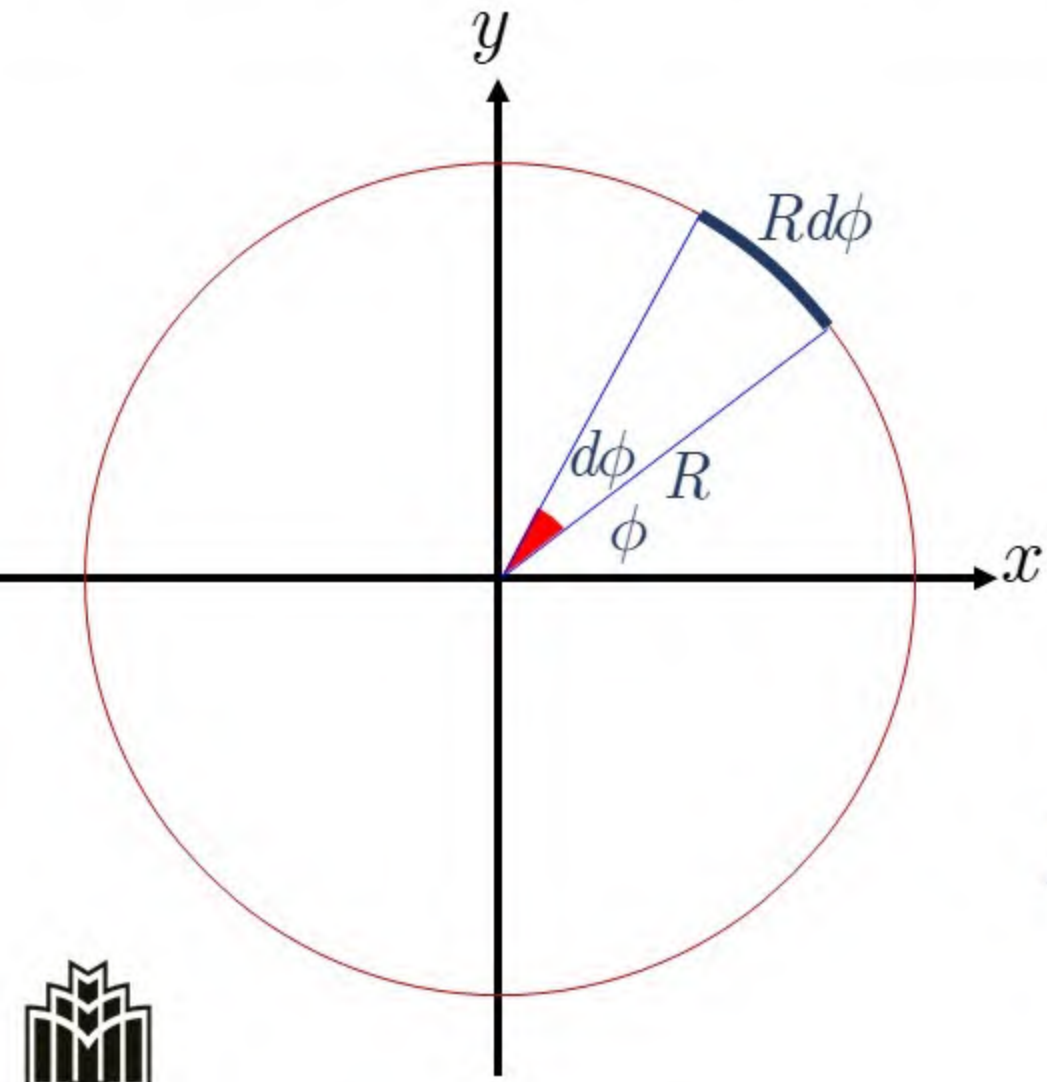
$$\begin{aligned} \cos\phi &= \frac{x}{\sqrt{x^2 + y^2}} \\ \sin\phi &= \frac{y}{\sqrt{x^2 + y^2}} \end{aligned}$$



$$\boldsymbol{A} = A_\rho \hat{\boldsymbol{e}}_\rho + A_\phi \hat{\boldsymbol{e}}_\phi$$

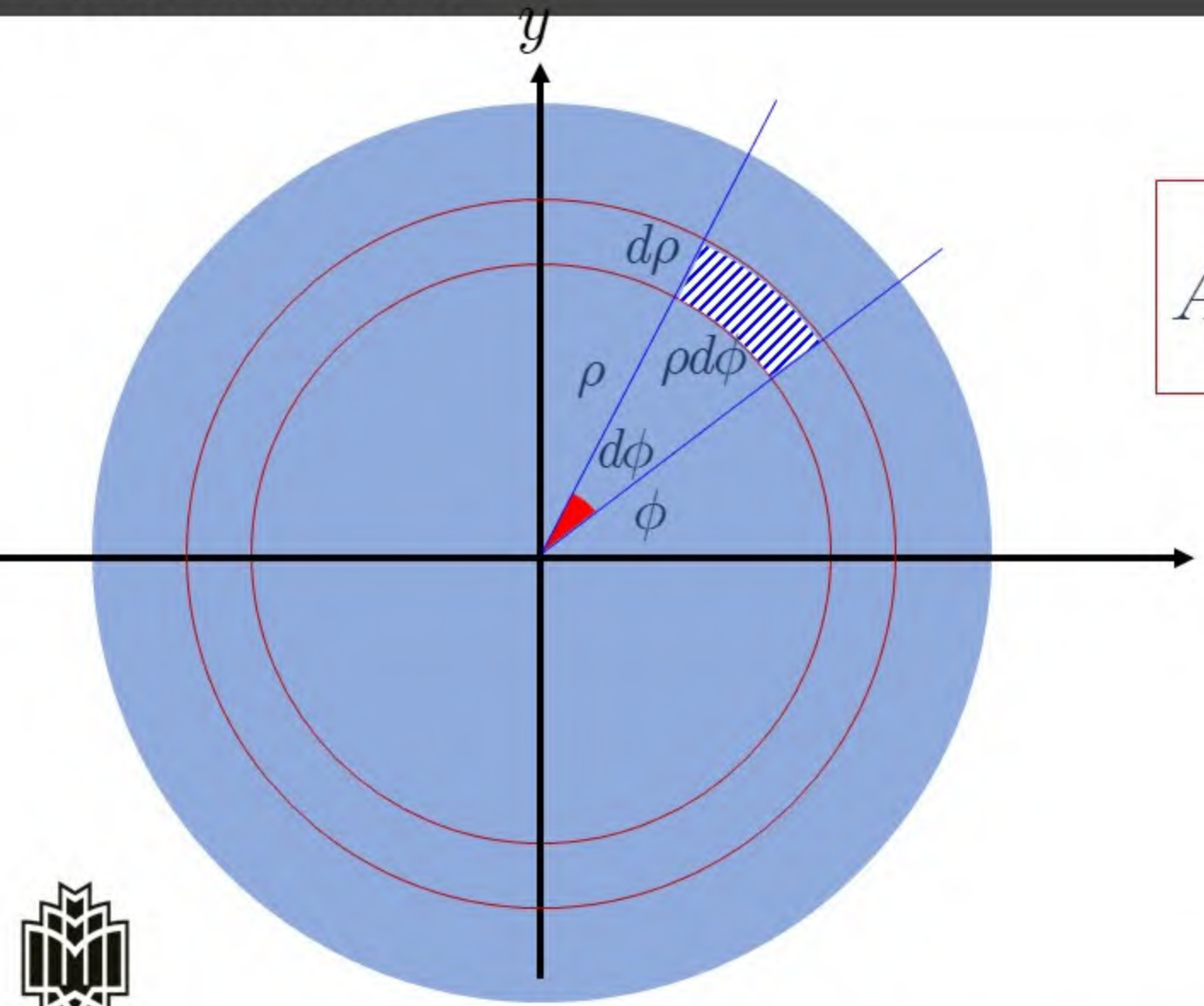


$$da = \rho d\rho d\phi$$



$$dl = Rd\phi$$

$$L = \int dl = R\int_0^{2\pi} d\phi = R\phi\Big|_0^{2\pi} = 2\pi R$$



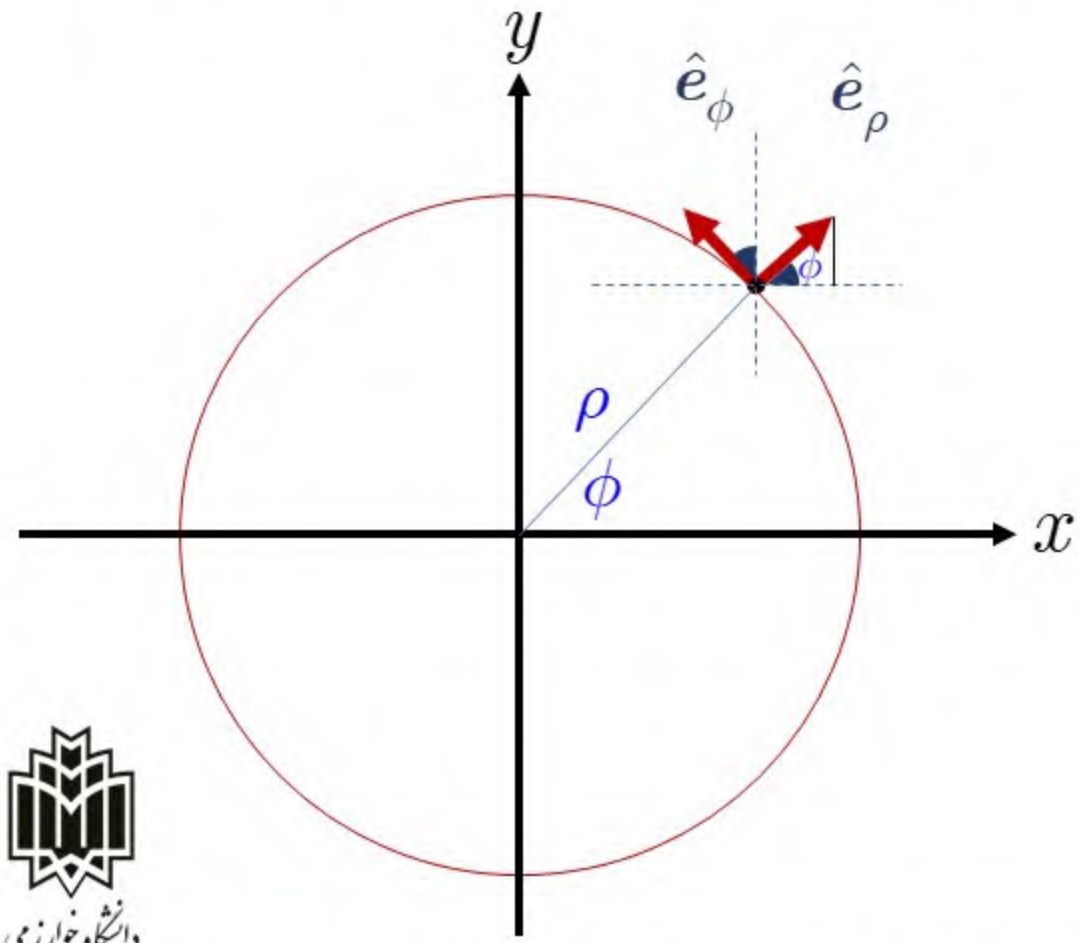
$$da = \rho d\rho d\phi$$

$$A = \int da = \int_0^{2\pi} \int_0^R \rho d\rho d\phi$$

$$A = \int_0^{2\pi} d\phi \left[\int_0^R \rho d\rho \right]$$

$$A = \left(2\pi\right)\left(\frac{1}{2}R^2\right) = \pi R^2$$

The relationship between polar and Cartesian coordinate systems



$$\hat{e}_\rho = \cos\phi\hat{e}_x + \sin\phi\hat{e}_y$$
$$\hat{e}_\phi = -\sin\phi\hat{e}_x + \cos\phi\hat{e}_y$$

$$\hat{e}_x = \cos\phi\hat{e}_\rho - \sin\phi\hat{e}_\phi$$
$$\hat{e}_y = \sin\phi\hat{e}_\rho + \cos\phi\hat{e}_\phi$$

$$A_x = \boldsymbol{A}\cdot\hat{e}_x = A_\rho\hat{e}_\rho\cdot\hat{e}_x + A_\phi\hat{e}_\phi\cdot\hat{e}_x$$

$$A_y = \boldsymbol{A}\cdot\hat{e}_y = A_\rho\hat{e}_\rho\cdot\hat{e}_y + A_\phi\hat{e}_\phi\cdot\hat{e}_y$$

$$A_x = A_\rho\cos\phi - A_\phi\sin\phi$$
$$A_y = A_\rho\sin\phi + A_\phi\cos\phi$$

$$\boxed{\begin{aligned} A_x &= A_\rho \cos\phi - A_\phi \sin\phi \\ A_y &= A_\rho \sin\phi + A_\phi \cos\phi \end{aligned}}$$

$$\boxed{\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_\rho \\ A_\phi \end{pmatrix}}$$

$$\boxed{\begin{pmatrix} A_\rho \\ A_\phi \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_x \\ A_y \end{pmatrix}}$$

$$\boxed{\begin{pmatrix} A_x \\ A_y \end{pmatrix} = \begin{pmatrix} \dfrac{x}{\sqrt{x^2+y^2}} & -\dfrac{y}{\sqrt{x^2+y^2}} \\ \dfrac{y}{\sqrt{x^2+y^2}} & \dfrac{x}{\sqrt{x^2+y^2}} \end{pmatrix} \begin{pmatrix} A_\rho \\ A_\phi \end{pmatrix}}$$

شاد و مهربان باشید

Be happy and kind