Kharazmi University

Faculty of Physics

شماره‌ی تکلیف: ۵

Problem 1:

The Cartesian coordinates of point A are $(3, 2, 1)$. Convert these coordinates to cylindrical coordinates.

Answer:

$$\left(\sqrt{13},\ \tan^{-1}\left(\frac{2}{3}\right),\ -1\right) = (3.606, 0.588, -1) = (3.606, 33.69^\circ, -1)$$

Problem 2:

The spherical coordinates of point P are $(1,\ \pi/4,\ \pi/2)$. Convert these coordinates to Cartesian coordinates.

Answer:

$$\left(0,\ \frac{\sqrt{2}}{2},\ \frac{\sqrt{2}}{2}\right)$$

Problem 3:

The vector $\boldsymbol{A} = 3\hat{e}_\rho + \hat{e}_\phi + 2\hat{e}_z$ is given in cylindrical coordinates. Convert this vector to Cartesian coordinates at the point $(2, \pi/4, -1)$.

Answer:

$$\boldsymbol{A} = \sqrt{2}\,\hat{\boldsymbol{e}}_x + 2\sqrt{2}\,\hat{\boldsymbol{e}}_y + 2\hat{\boldsymbol{e}}_z$$

Problem 4:

The vector field $\boldsymbol{F} = \frac{x\,\hat{\boldsymbol{e}}_x + y\,\hat{\boldsymbol{e}}_y + 4\,\hat{\boldsymbol{e}}_z}{\sqrt{x^2+y^2+z^2}}$ is given in Cartesian coordinates. Convert $\boldsymbol{F}$ to spherical coordinates.

Answer:

$$\boldsymbol{F} = \left(\sin^2\theta + \frac{4\cos\theta}{r}\right)\hat{\boldsymbol{e}}_r + \left(\sin\theta\cos\theta - \frac{4\sin\theta}{r}\right)\hat{\boldsymbol{e}}_\theta$$

Problem 5:

Show that the transformation relationship between vector components in spherical (r, θ, ϕ) and cylindrical (ρ, ϕ, z) coordinates is given by:

$$\begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix} = \begin{bmatrix} \sin\theta & 0 & \cos\theta \\ \cos\theta & 0 & -\sin\theta \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix}.$$

Also, derive the inverse transformation (spherical to cylindrical coordinates).

Answer:

$$\begin{bmatrix} A_\rho \\ A_\phi \\ A_z \end{bmatrix} = \begin{bmatrix} \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \\ \cos\theta & -\sin\theta & 0 \end{bmatrix} \begin{bmatrix} A_r \\ A_\theta \\ A_\phi \end{bmatrix}.$$