شماره‌ی تکلیف: ۳

یادآوری: نماد کرونکر δ_{ij} و نماد لِوی‌ـچیویتا ϵ_{ijk} به شکل زیر تعریف می‌شوند:

$$\delta_{ij} = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases}$$

$$\epsilon_{ijk} = \begin{cases} 1 & ijk = 123,\ 231,\ 312 \\ 0 & \text{در صورتی که حداقل دو اندیس برابر باشند} \\ -1 & ijk = 132,\ 213,\ 321 \end{cases}$$

می‌توان نماد لوی‌چیویتا را به شکل ساده‌ی زیر نیز بیان کرد:

$$\epsilon_{ijk} = \frac{1}{2}(i-j)(j-k)(k-i)$$

Problem 1:

Calculate the value of each of the following expressions.

☞ (a) $\sum_{i=1}^{3} \delta_{ii} = ?$

☞ (b) $\sum_{i=1}^{3}\sum_{j=1}^{3} \delta_{ij}\epsilon_{ijk} = ?$

☞ (c) $\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon_{ijk}\epsilon_{mjk} = ?$

☞ (d) $\sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon_{ijk}\epsilon_{ijk} = ?$

Problem 2:

Show that

$$\hat{\boldsymbol{e}}_i \cdot (\hat{\boldsymbol{e}}_j \times \hat{\boldsymbol{e}}_k) = \epsilon_{ijk}$$

where $\hat{\boldsymbol{e}}_n$ $(n = 1, 2, 3)$ are the orthonormal right-handed basis vectors in the order $1, 2, 3$.

Problem 3:

Show that

$$\boldsymbol{A} \times \boldsymbol{B} = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon_{ijk} A_i B_j \hat{\boldsymbol{e}}_k$$

Problem 4:

Show that

$$\boldsymbol{A} \cdot (\boldsymbol{B} \times \boldsymbol{C}) = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon_{ijk} A_i B_j C_k$$

Problem 5:

Show that

$$\boldsymbol{A} \cdot (\boldsymbol{B} \times \boldsymbol{C}) = (\boldsymbol{A} \times \boldsymbol{B}) \cdot \boldsymbol{C}$$

Problem 6:

Show that

$$\sum_{i=1}^{3} \epsilon_{ijk}\epsilon_{ilm} = \delta_{jl}\delta_{km} - \delta_{jm}\delta_{kl}$$

Problem 7:

Show that

$$\sum_{j=1}^{3}\sum_{k=1}^{3} \epsilon_{ijkmjk} = 2\delta_{im}$$

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Problem 8:

Show that $\boldsymbol{A} \times (\boldsymbol{B} \times \boldsymbol{C}) = \boldsymbol{B}(\boldsymbol{A} \cdot \boldsymbol{C}) - \boldsymbol{C}(\boldsymbol{A} \cdot \boldsymbol{B})$. This is known as the BAC-CAB rule.