شماره‌ی تکلیف: ۲

Problem 1:

Given the vectors:

$$\boldsymbol{A} = 4.0\hat{\boldsymbol{e}}_x - 3.0\hat{\boldsymbol{e}}_y + 1.0\hat{\boldsymbol{e}}_z$$

and

$$\boldsymbol{B} = -1.0\hat{\boldsymbol{e}}_x + 1.0\hat{\boldsymbol{e}}_y + 4.0\hat{\boldsymbol{e}}_z,$$

perform the following operations:

☞ (a) Express the vector $\boldsymbol{A} - \boldsymbol{B}$ in terms of unit vectors.

☞ (b) Express the vector $\boldsymbol{A} + \boldsymbol{B}$ in terms of unit vectors.

☞ (c) Determine the vector $\boldsymbol{C}$ such that $\boldsymbol{A} - \boldsymbol{B} + \boldsymbol{C} = 0$.

Problem 2:

Three vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{C}$ lie in the xy-plane. The magnitudes of these vectors are equal and are given by $50\ \mathrm{m}$. The angles of each vector relative to the x-axis are $30°$, $195°$, and $315°$, respectively.

☞ (a) Determine the magnitude and direction (angle relative to the x-axis) of the vector $\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C}$.

☞ (b) If a fourth vector $\boldsymbol{D}$ is such that $(\boldsymbol{A} + \boldsymbol{B}) - (\boldsymbol{C} + \boldsymbol{D}) = 0$, determine its magnitude and direction.

Problem 3:

Determine the magnitude of the vector $\boldsymbol{B}$ using the following information:
When the vector $\boldsymbol{B}$ is added to the vector $\boldsymbol{C} = 3.0\hat{\boldsymbol{e}}_x + 4.0\hat{\boldsymbol{e}}_y$, the resultant vector lies entirely in the positive y-direction and has a magnitude equal to that of $\boldsymbol{C}$.

Problem 4:

The following relations hold between the vectors $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$:

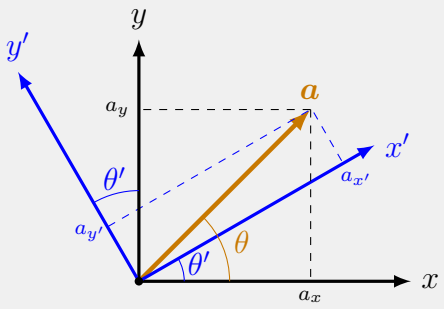
$$\boldsymbol{A} - \boldsymbol{B} = 3\boldsymbol{C}$$

$$\boldsymbol{A} + \boldsymbol{B} = 5\boldsymbol{C}$$

If $\boldsymbol{C} = 2\hat{\boldsymbol{e}}_x + 4\hat{\boldsymbol{e}}_y$, determine the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$.

Problem 5:

The magnitude of the vector $\boldsymbol{a}$ is $17.0\ \mathrm{m}$, and it makes an angle of $\theta = 56.0^\circ$ with the $+x$-axis.



☞ (a) Determine the components a_x and a_y of the vector $\boldsymbol{a}$.

☞ (b) If the coordinate system is rotated counterclockwise about the origin by an angle of $\theta' = 18.0^\circ$, determine the components $a_{x'}$ and $a_{y'}$ in the new coordinate system.

Problem 6:

The vectors $\boldsymbol{A} = 3.0\hat{\boldsymbol{e}}_x + 5.0\hat{\boldsymbol{e}}_y$ and $\boldsymbol{B} = 2.0\hat{\boldsymbol{e}}_x + 4.0\hat{\boldsymbol{e}}_y$ are given.

☞ (a) Find $\boldsymbol{A} \times \boldsymbol{B}$.

☞ (b) Find $\boldsymbol{A} \cdot \boldsymbol{B}$.

☞ (c) Find $(\boldsymbol{A} + \boldsymbol{B}) \cdot \boldsymbol{B}$.

☞ (d) Find the projection of vector $\boldsymbol{A}$ onto vector $\boldsymbol{B}$.

Problem 7:

The vectors

$$\boldsymbol{A} = 2.00\hat{\boldsymbol{e}}_x + 3.00\hat{\boldsymbol{e}}_y - 4.00\hat{\boldsymbol{e}}_z, \boldsymbol{B} = -3.00\hat{\boldsymbol{e}}_x + 4.00\hat{\boldsymbol{e}}_y + 2.00\hat{\boldsymbol{e}}_z, \boldsymbol{C} = 7.00\hat{\boldsymbol{e}}_x - 8.00\hat{\boldsymbol{e}}_y$$

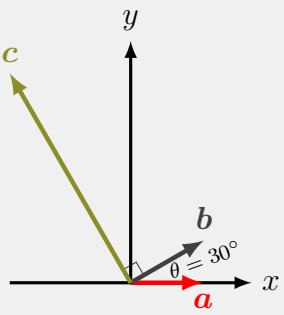
are given.
Find the value of the expression $3\boldsymbol{C} \cdot (2\boldsymbol{A} \times \boldsymbol{B})$.

Problem 8:

The vectors illustrated in the figure have the following magnitudes:

$$|\boldsymbol{a}| = 3.00\ \text{m}, \quad |\boldsymbol{b}| = 4.00\ \text{m}, \quad |\boldsymbol{c}| = 10.0\ \text{m},$$

and the angle θ is 30°.



☞ (a) Determine the x- and y-components of each of these vectors.

☞ (b) If the relationship $\boldsymbol{c} = p\boldsymbol{a} + q\boldsymbol{b}$ holds, determine the values of p and q.

Problem 9:

In the relation $\boldsymbol{C} = a\boldsymbol{A} \times \boldsymbol{B}$, assume $a = 2$, $\boldsymbol{A} = 2.0\hat{\boldsymbol{e}}_x + 4.0\hat{\boldsymbol{e}}_y + 6.0\hat{\boldsymbol{e}}_z$, and $\boldsymbol{C} = 4.0\hat{\boldsymbol{e}}_x - 20.0\hat{\boldsymbol{e}}_y + 12.0\hat{\boldsymbol{e}}_z$. If $B_x = B_y$, express the vector $\boldsymbol{B}$ in terms of unit vectors.

Problem 10:

The magnitude of the vector $\boldsymbol{A}$ is $12.0\ \text{m}$, and its angle relative to the $+x$-axis in the two-dimensional xy-coordinate system is 60.0°. In the same coordinate system, the vector $\boldsymbol{B}$ is given by

$$\boldsymbol{B} = (12.0\ \text{m})\hat{\boldsymbol{e}}_x + (8.00\ \text{m})\hat{\boldsymbol{e}}_y.$$

Now, the coordinate system is rotated counterclockwise about the origin by 20.0° to obtain a new coordinate system $x'y'$.

In this new coordinate system, express the components of the vectors $\boldsymbol{A}$ and $\boldsymbol{B}$ in terms of the unit vectors.