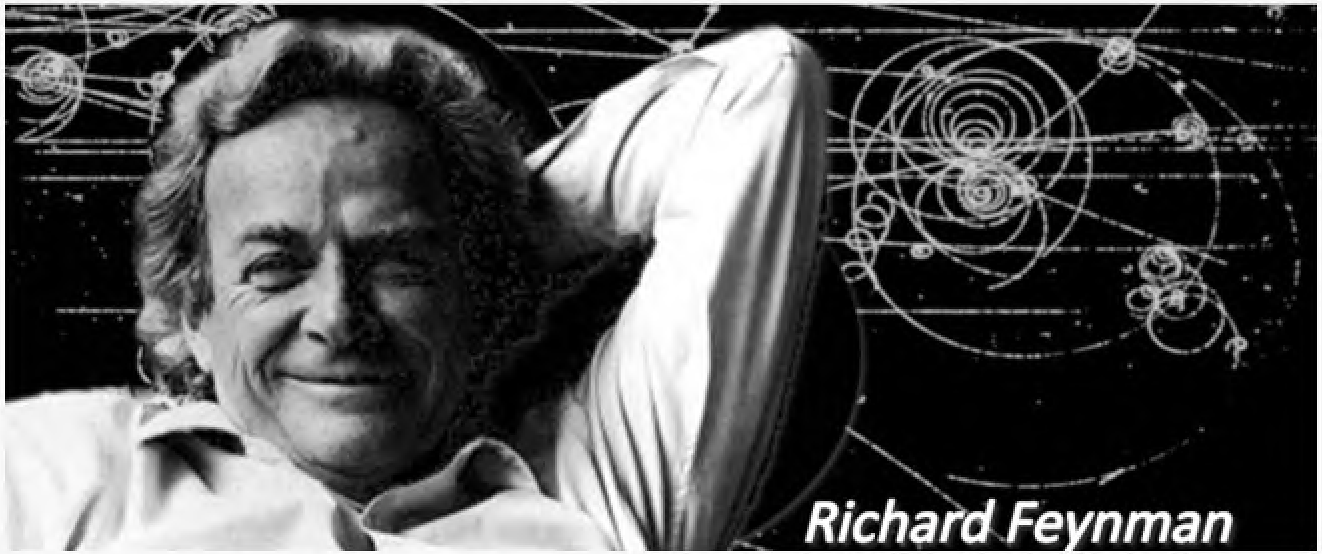
Electromagnetism I

Faculty of Physics-Kharazmi University

Dr. Faramarz Kanjouri



اگر همواره مانند گذشته بیندیشید، همیشه همان چیزهایی را به‌دست می‌آورید که تا کنون کسب کرده‌اید

فاینمن

درس سی و هفتم

مغناطوستاتیک بخش سوم

MagnetoStatics-part3

دانشگاه خوارزمی

برای جریان‌های پایا دیدیم:

$$\nabla \cdot \vec{J}(\vec{r}) = 0$$

شکل دیفرانسیلی قانون آمپر

$$\nabla \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$

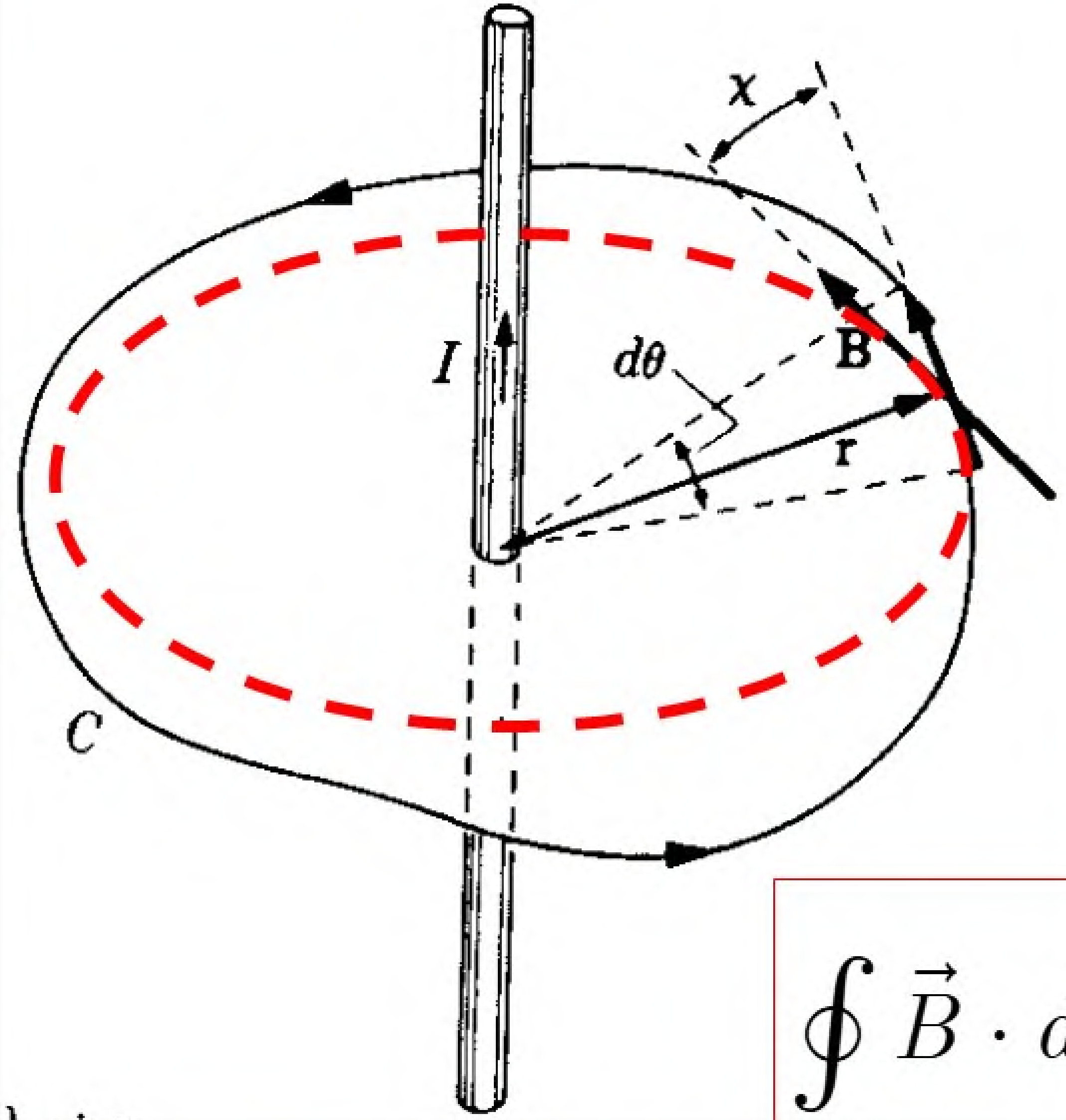
قضیه‌ی استوکس

$$\int_S \nabla \times \vec{B} \cdot \hat{n} da = \oint_C \vec{B} \cdot d\vec{l}$$

$$\int_S \mu_0 \vec{J} \cdot \hat{n} da = \oint_C \vec{B} \cdot d\vec{l}$$

$$\boxed{\mu_0 I = \oint_C \vec{B} \cdot d\vec{l}}$$

بررسی درستی قانون آمپر برای یک سیم مستقیم بسیار بلند حامل جریان الکتریکی I



$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$$\vec{B}\cdot d\vec{l} = B\,dl\cos\chi$$

$$\vec{B}\cdot d\vec{l} = B\,dl\cos\chi = B\,r\,d\theta$$

$$\oint \vec{B}\cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi r} r\,d\theta = \frac{\mu_0 I}{2\pi}\oint d\theta = \mu_0 I$$

$$\boxed{\nabla \cdot \vec{B} = 0 \quad \rightarrow \quad \vec{B} = \nabla \times \vec{A}}$$

$$\boxed{\vec{A}' = \vec{A} + \nabla\Psi}$$

$$\boxed{\nabla \times \vec{A}' = \nabla \times \vec{A} + \nabla \times \nabla\Psi = \nabla \times \vec{A} = \vec{B}}$$

$$\nabla \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$

$$\nabla \times \nabla \times \vec{A}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$

$$\nabla \times \nabla \times \vec{A}(\vec{r}) = \nabla\nabla \cdot \vec{A}(\vec{r}) - \nabla^2 \vec{A}(\vec{r})$$

$$\nabla\nabla \cdot \vec{A}(\vec{r}) - \nabla^2 \vec{A}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$

$$\nabla \cdot \vec{A}(\vec{r}) = 0$$ پیمانه‌ی کولن

$$\nabla^2 \vec{A}(\vec{r}) = -\mu_0 \vec{J}(\vec{r})$$

$$\nabla^2 \vec{A}(\vec{r}) = \hat{i}\nabla^2 A_x(\vec{r}) + \hat{j}\nabla^2 A_y(\vec{r}) + \hat{k}\nabla^2 A_z(\vec{r})$$

$$\nabla^2 \vec{A}(\vec{r}) = -\mu_0 \vec{J}(\vec{r})$$

$$\nabla^2 A_x(\vec{r}) = \mu_0 J_x(\vec{r})$$
$$\nabla^2 A_y(\vec{r}) = \mu_0 J_y(\vec{r})$$
$$\nabla^2 A_z(\vec{r}) = \mu_0 J_z(\vec{r})$$

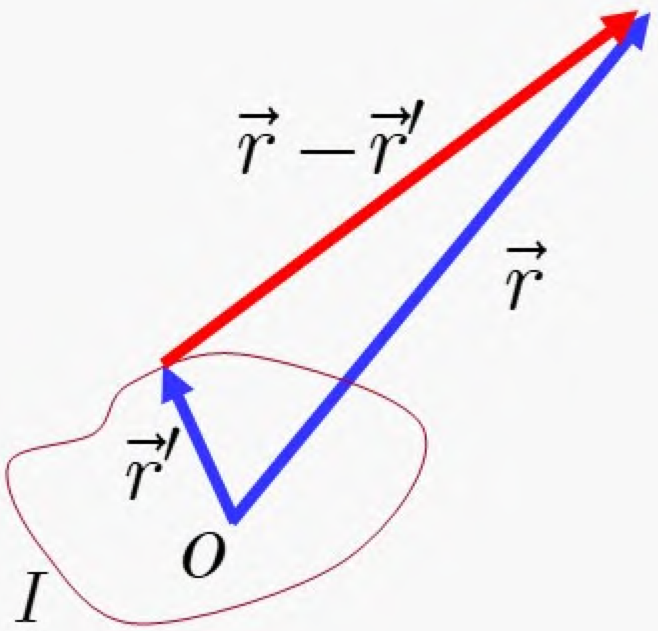
مقایسه با الکتروستاتیک:

$$\nabla^2 \Phi(\vec{r}) = \frac{1}{\varepsilon_0}\rho(\vec{r}) \quad \rightarrow \quad \Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}\,')dv'}{|\vec{r} - \vec{r}\,'|}$$

$$\nabla^2 A_x(\vec{r}) = \mu_0 J_x(\vec{r})$$
$$\nabla^2 A_y(\vec{r}) = \mu_0 J_y(\vec{r})$$
$$\nabla^2 A_z(\vec{r}) = \mu_0 J_z(\vec{r})$$

$$A_x(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{J_x(\vec{r}')dv'}{|\vec{r} - \vec{r}'|}$$
$$A_y(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{J_y(\vec{r}')dv'}{|\vec{r} - \vec{r}'|}$$
$$A_z(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{J_z(\vec{r}')dv'}{|\vec{r} - \vec{r}'|}$$

$$\boxed{\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}')dv'}{|\vec{r} - \vec{r}'|}}$$



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\int \frac{I d\vec{l}'}{|\vec{r}-\vec{r}'|}$$

$$d\vec{l}' \equiv d\vec{r}'$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{\sqrt{r^2 - 2\vec{r}\cdot\vec{r}' + r'^2}}$$

$$\frac{1}{|\vec{r}-\vec{r}'|} = \frac{1}{r}\left[1 + \frac{\vec{r}\cdot\vec{r}'}{r^2}\cdots\right]$$

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\left[\frac{1}{r}\oint d\vec{r}' + \frac{1}{r^3}\oint d\vec{r}'(\vec{r}\cdot\vec{r}') + \cdots\right]$$

$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\left[\frac{1}{r}\cancelto{0}{\oint d\vec{r}\,'} + \frac{1}{r^3}\oint d\vec{r}\,'(\vec{r}\cdot\vec{r}\,') + \cdots\right]$$

$$\left(\vec{r}\,' \times d\vec{r}\,'\right) \times \vec{r} = d\vec{r}\,'\left(\vec{r}\cdot\vec{r}\,'\right) - \vec{r}\,'\left(\vec{r}\cdot d\vec{r}\,'\right)$$

$$d\left[\vec{r}\,'\left(\vec{r}\cdot\vec{r}\,'\right)\right] = \vec{r}\,'\left(\vec{r}\cdot d\vec{r}\,'\right) + d\vec{r}\,'\left(\vec{r}\cdot\vec{r}\,'\right)$$

دو رابطه‌ی فوق را با هم جمع کنید:

$$d\vec{r}\,'\left(\vec{r}\cdot\vec{r}\,'\right) = \frac{1}{2}\left(\vec{r}\,' \times d\vec{r}\,'\right) \times \vec{r} + \frac{1}{2} d\left[\vec{r}\,'\left(\vec{r}\cdot\vec{r}\,'\right)\right]$$

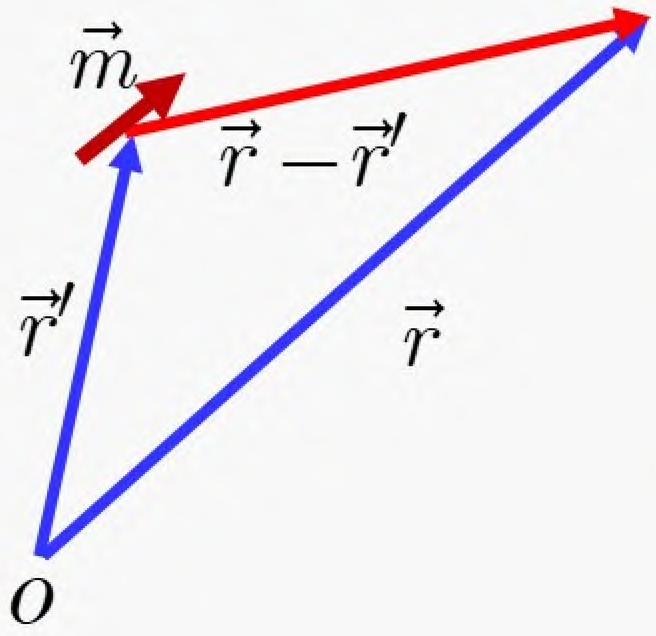
$$\vec{A}(\vec{r}) = \frac{\mu_0 I}{4\pi}\frac{1}{r^3}\oint d\vec{r}'(\vec{r}\cdot\vec{r}')$$

$$d\vec{r}'\left(\vec{r}\cdot\vec{r}'\right) = \frac{1}{2}\left(\vec{r}'\times d\vec{r}'\right)\times\vec{r} + \frac{1}{2}d\left[\vec{r}'\left(\vec{r}\cdot\vec{r}'\right)\right]$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\left[\frac{I}{2}\oint \vec{r}'\times d\vec{r}'\right]\times\frac{\vec{r}}{r^3}$$

$$\vec{m} = \frac{I}{2}\oint \vec{r}'\times d\vec{r}'$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\vec{m}\times\frac{\vec{r}}{r^3}$$



$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^3}$$

$$\vec{B} = \nabla \times \vec{A}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m} \times \left(\vec{r} - \vec{r}'\right)}{\left|\vec{r} - \vec{r}'\right|^3}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\nabla \times \left[\frac{\vec{m} \times \left(\vec{r} - \vec{r}'\right)}{\left|\vec{r} - \vec{r}'\right|^3}\right]$$

$$4\pi\delta\left(\vec{r} - \vec{r}'\right)$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left[-\left(\vec{m} \cdot \nabla\right)\frac{\left(\vec{r} - \vec{r}'\right)}{\left|\vec{r} - \vec{r}'\right|^3} + \vec{m}\nabla \cdot \frac{\left(\vec{r} - \vec{r}'\right)}{\left|\vec{r} - \vec{r}'\right|^3}\right]$$

دانشگاه خوارزمی

$$\vec{B}=\frac{\mu_0}{4\pi}\left[-\left(\vec{m}\cdot\nabla\right)\frac{\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^3}\right]$$

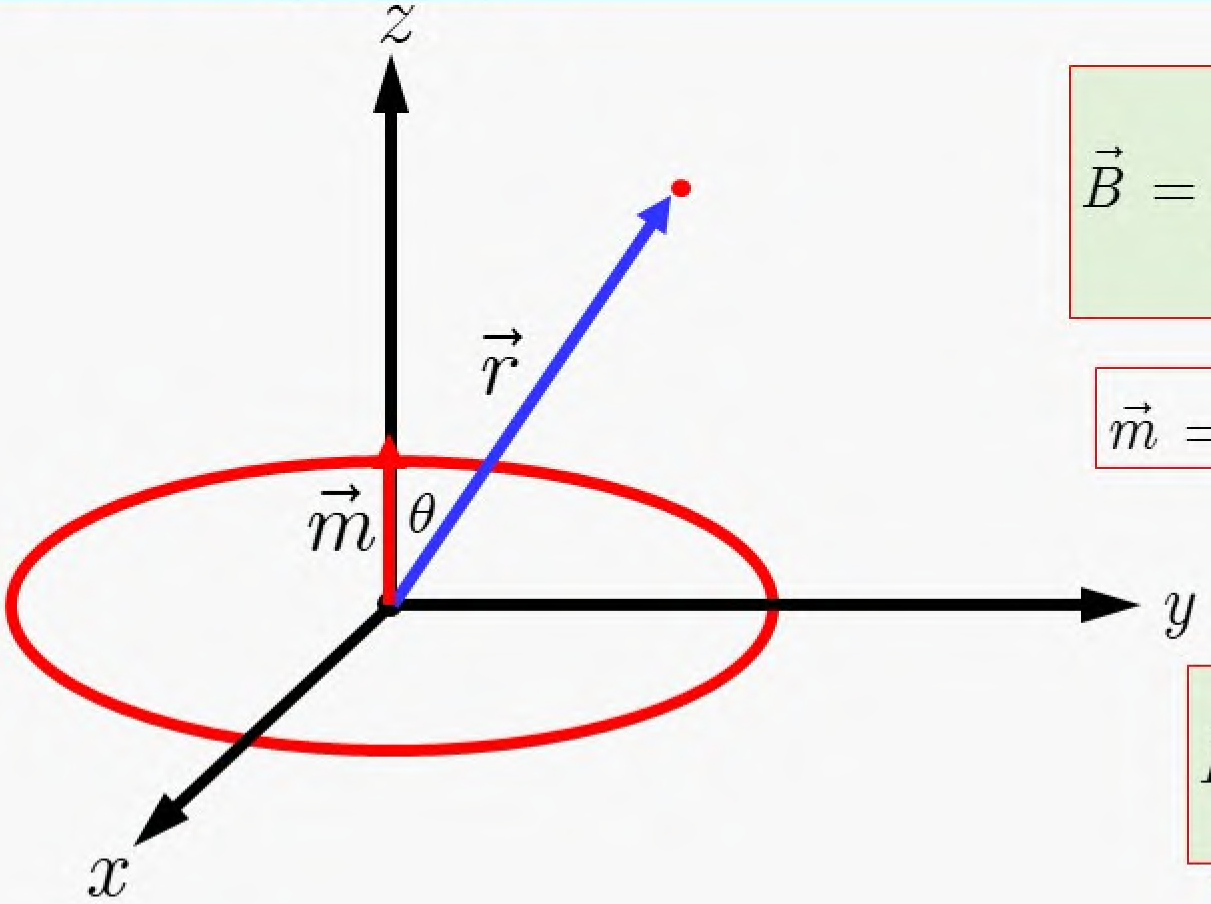
$$\left(\vec{m}\cdot\nabla\right)\frac{\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^3}=m_x\frac{\partial}{\partial x}\frac{\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^3}+m_y\frac{\partial}{\partial y}\frac{\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^3}+m_z\frac{\partial}{\partial z}\frac{\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^3}$$

$$m_x\frac{\partial}{\partial x}\frac{\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^3}=\frac{m_x\hat{i}}{\left|\vec{r}-\vec{r}'\right|^3}-3m_x\left(x-x'\right)\frac{\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^5}$$

$$\vec{B}=\frac{\mu_0}{4\pi}\left[-\frac{\vec{m}}{\left|\vec{r}-\vec{r}'\right|^3}+\frac{3\vec{m}\cdot\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^5}\left(\vec{r}-\vec{r}'\right)\right]$$

میدان مغناطیسی یک دوقطبی مغناطیسی

دانشگاه خوارزمی



$$\vec{B} = \frac{\mu_0}{4\pi}\left[-\frac{\vec{m}}{\left|\vec{r}-\vec{r}'\right|^3} + \frac{3\vec{m}\cdot\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^5}\left(\vec{r}-\vec{r}'\right)\right]$$

$$\vec{m} = m\hat{k} = I\pi R^2\hat{k} \qquad R \ll r \qquad \vec{r}' = 0$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left[-\frac{m\hat{k}}{r^3} + \frac{3m\cos\theta}{r^3}\hat{r}\right]$$

$$\hat{k} = \hat{r}\cos\theta - \hat{\theta}\sin\theta$$

$$\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\hat{r}\cos\theta + \hat{\theta}\sin\theta\right)$$

دانشگاه خوارزمی

$$\nabla \times \vec{B}(\vec{r}) = \mu_0 \vec{J}(\vec{r})$$ شکل دیفرانسیلی قانون آمپر

برای نقاطی که چگالی جریان صفر است $\rightarrow \quad \vec{J}(\vec{r}) = 0 \quad \rightarrow \quad \nabla \times \vec{B}(\vec{r}) = 0$

$$\vec{B}(\vec{r}) = -\mu_0 \nabla \Phi^*(\vec{r})$$

$$\nabla \cdot \vec{B}(\vec{r}) = 0 \quad \Rightarrow \quad \nabla \cdot \vec{B}(\vec{r}) = -\mu_0 \nabla^2 \Phi^*(\vec{r}) = 0$$

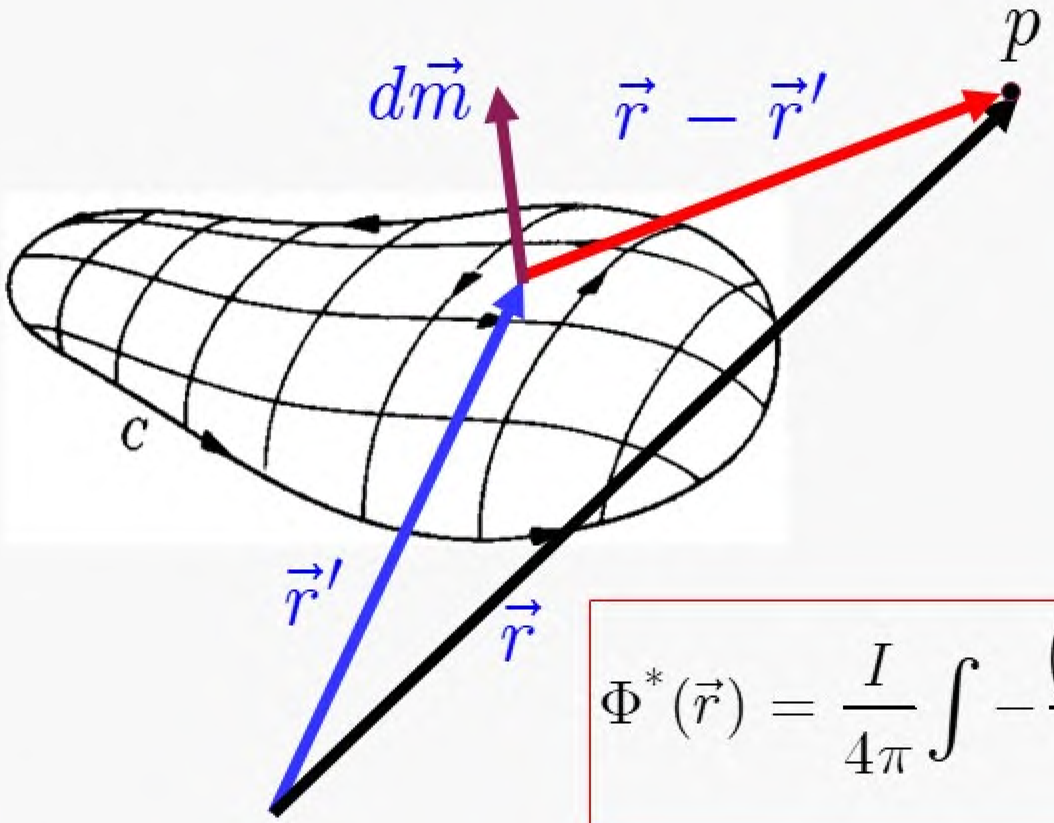
$$\nabla^2 \Phi^*(\vec{r}) = 0$$

$$\vec{B} = \frac{\mu_0}{4\pi}\left[-\frac{\vec{m}}{\left|\vec{r}-\vec{r}'\right|^3} + \frac{3\vec{m}\cdot\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^5}\left(\vec{r}-\vec{r}'\right)\right]$$

$$\vec{B} = -\frac{\mu_0}{4\pi}\nabla\left[\frac{\vec{m}\cdot\left(\vec{r}-\vec{r}'\right)}{\left|\vec{r}-\vec{r}'\right|^3}\right]$$

$$\Phi^*(\vec{r}) = \frac{\vec{m}\cdot\left(\vec{r}-\vec{r}'\right)}{4\pi\left|\vec{r}-\vec{r}'\right|^3}$$

پتانسیل نرده‌ای یک دوقطبی مغناطیسی



$$d\vec{m} = I\hat{n}da'$$

$$d\Phi^*(\vec{r}) = \frac{d\vec{m}\cdot(\vec{r}-\vec{r}')}{4\pi|\vec{r}-\vec{r}'|^3}$$

$$\Phi^*(\vec{r}) = \frac{I}{4\pi}\int\frac{(\vec{r}-\vec{r}')\cdot\hat{n}da'}{|\vec{r}-\vec{r}'|^3}$$

$$\Phi^*(\vec{r}) = \frac{I}{4\pi}\int -\frac{(\vec{r}'-\vec{r})\cdot\hat{n}da'}{|\vec{r}-\vec{r}'|^3}$$

$$\Phi^*(\vec{r}) = -\frac{I}{4\pi}\int d\Omega_p$$

$$\boxed{\Phi^*(\vec{r}) = -\frac{I}{4\pi}\Omega_p}$$

$$\Phi = \int_S \vec{B} \cdot \hat{n} da$$ شار مغناطیسی برحسب وبر Wb

$$\oint \vec{B} \cdot \hat{n} da = \int \nabla \cdot \vec{B} dv = 0$$

دانشگاه خوارزمی

شاد و مهربان باشید